Modeling Lateral Charge Dynamics and Space-Charge Screening in Ionization Chambers under UHDR Proton Pencil Beam

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Purpose:

Ultra-High Dose Rate (UHDR) FLASH radiotherapy can potentially widen the therapeutic window by reducing normal tissue radiotoxicities while maintaining equipotent tumor control. However, the safe implementation of FLASH trials is currently hindered by a lack of traceable dosimetry standards and inter-institutional protocols. Conventional ionization chambers suffer from severe ion recombination losses at UHDR intensities. While analytical theories successfully correct for recombination in clinical beams, their application under FLASH conditions where high charge densities modify the underlying electric field must be revisited. Under UHDR where ions are liberated in large quantities, the independent particle approximation fails. The high-density ion cloud creates recombination and self-shielding effects that effectively couple the transport of positive and negative species. We have performed recombination measurements of isochronous cyclotron-generated UHDR proton beams using a large-area transmission IC, with a Faraday Cup (FC) serving as the recombination-independent baseline. In parallel, we used a 1D numerical model solving drift-diffusion-reaction equations to characterize the ion recombination effects. However, 1D approximations are geometrically limited; a multi-dimensional approach better approximates the charge dynamics in the ion chamber resulting from non-uniform irradiation beam and electric field. This work presents a novel 2D axisymmetric Finite Element Analysis (FEA) solver designed to handle these complex geometries. With a robust 2D geometric solver, we provide a theoretical framework for developing next-generation UHDR correction factors for clinically relevant detectors.

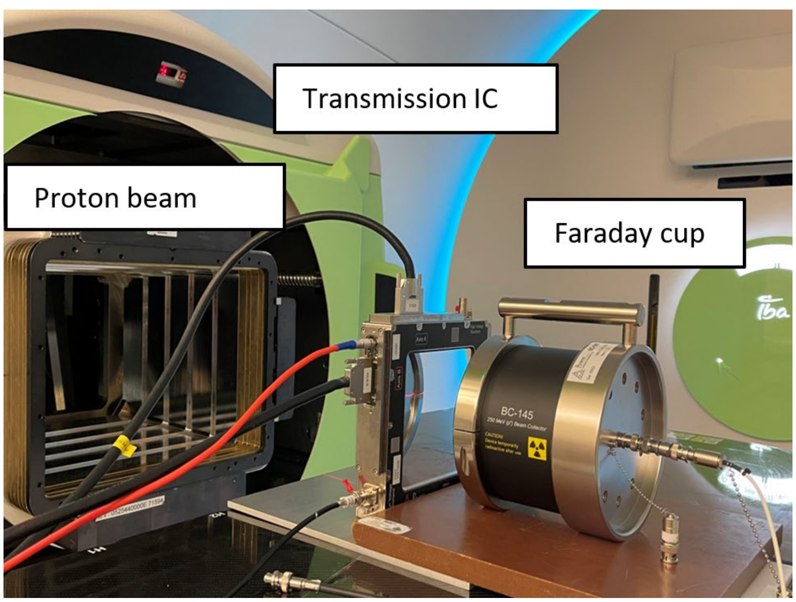

Figure 1. Measurement setup, showing the IC64 and BC145 aligned with the proton beam.

Methods:

Recombination was measured using a large area ionization chamber (IC64, Pyramid Technical Consultants) placed proximal to a Faraday Cup (BC145). The system was irradiated by a 230MeV quasi-continuous cyclotron proton beam (IBA Proteus Plus) at UHDR currents up to 500nA (Figure 1). The ionization chamber bias voltage was swept between 25V and 400V. The Faraday Cup provided recombination independent charge normalization, defining our experimental recombination ratio as $\tau = \frac{IC}{FC}$.

To model the internal chamber dynamics, the spatiotemporal evolution of positive ions (ρ_+), negative ions (ρ_-) and electrons (ρ_e) was described using a system of coupled drift-diffusion-reaction partial differential equations:

$$\frac{\partial \rho_+}{\partial t} = R(t) \qquad - \frac{\alpha}{e}\rho_+\rho_- - \nabla \cdot (v_+\rho_+)$$

$$\frac{\partial \rho_-}{\partial t} = \qquad + \gamma\rho_e - \frac{\alpha}{e}\rho_+\rho_- - \nabla \cdot (v_-\rho_-)$$

$$\frac{\partial \rho_e}{\partial t} = R(t) - \gamma\rho_e \qquad - \nabla \cdot (v_e\rho_e)$$

Here, $R(t)$ denotes the liberation of initial ion pairs, γ is the electron attachment rate, and $\frac{\alpha}{e}$ is the ion recombination rate. The charge velocities depend nonlinearly on the underlying electric field E, and collected charge was calculated according to the Shockley-Ramo theorem $I_{col} = qv \cdot E_w$ where E_w is the weighting field.

For initial validation of model dynamics, a 1D approximation was utilized. To account for the spatial inhomogeneity of a Gaussian proton pencil beam with maximum areal current density I_n, a Lebesgue slicing technique was developed. Assuming minimal lateral interactions, the current domain can be partitioned into n slices yielding the integrated recombination formula $\tau = \frac{1}{I_n}\sum_{j=1}^{n} \quad \tau_j(I_j - I_{j-1})$.

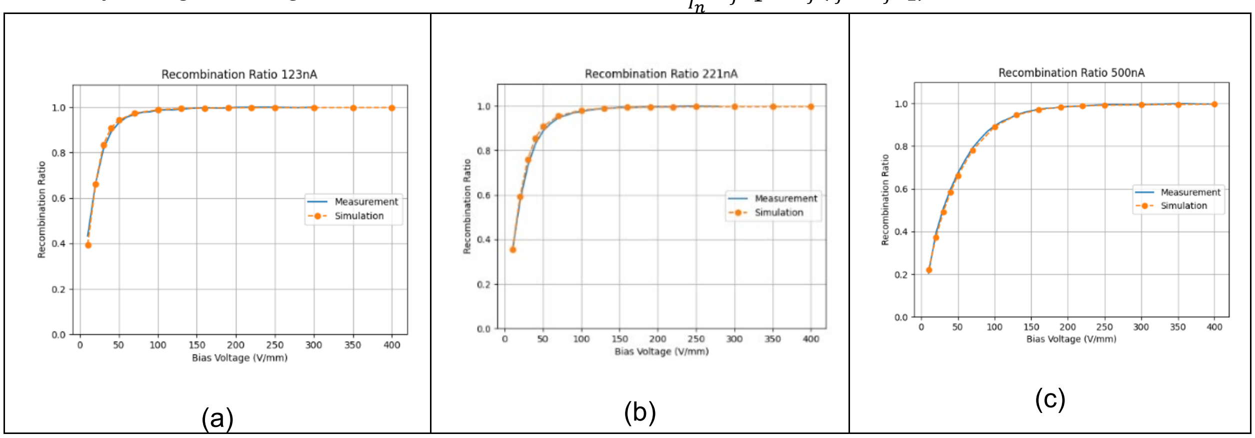

Figure 2. Measured and simulated recombination results at (a) 123nA proton current with a pencil beam with $\sigma = 5$ mm (b) 221nA proton current with a pencil beam with $\sigma = 5$ mm and (c) 500nA proton current with a pencil beam with $\sigma = 5.6$ mm. Measured results are shown in blue, with Lebesgue sliced simulation data shown in orange.

To capture complex multidimensional phenomena such as lateral diffusion and local field disturbance, a novel 2D axisymmetric solver was programmed in Julia. We employed a hybrid approach, with a Finite Element Analysis (FEA) solver evaluating the elliptic Poisson equation, while a coupled Finite Difference (FD) upwind scheme managed the hyperbolic advection-reaction transport. This approach guarantees excellent numerical stability for hyperbolic terms. To overcome the computational bottleneck of solving the electric field at sub-nanosecond temporal steps, the FEA solver relies on pre-computed Cholesky factorizations. Furthermore, by leveraging Julia's multi-threading libraries, avoiding type instability, and minimizing memory allocation overhead, the code achieves an order-of-magnitude (>10.8x) speedup over sequential baselines when benchmarked on a dual-socket Intel Xeon E5 system.

Results:

The experimental setup successfully characterized the recombination ratios across the measured bias voltage range for a UHDR proton pencil beam with sigma of 5 and 5.6 mm. At 100V, collection efficiency dropped from 98.69% at 123nA to 89.53% at 500nA. This falls well outside the clinical 5% target and highlights the inadequacy of clinical dosimetry models for UHDR beams. Applying the Lebesgue slicing technique yielded excellent quantitative agreement between the measured data and the 1D model across all tested configurations (Figure 2).

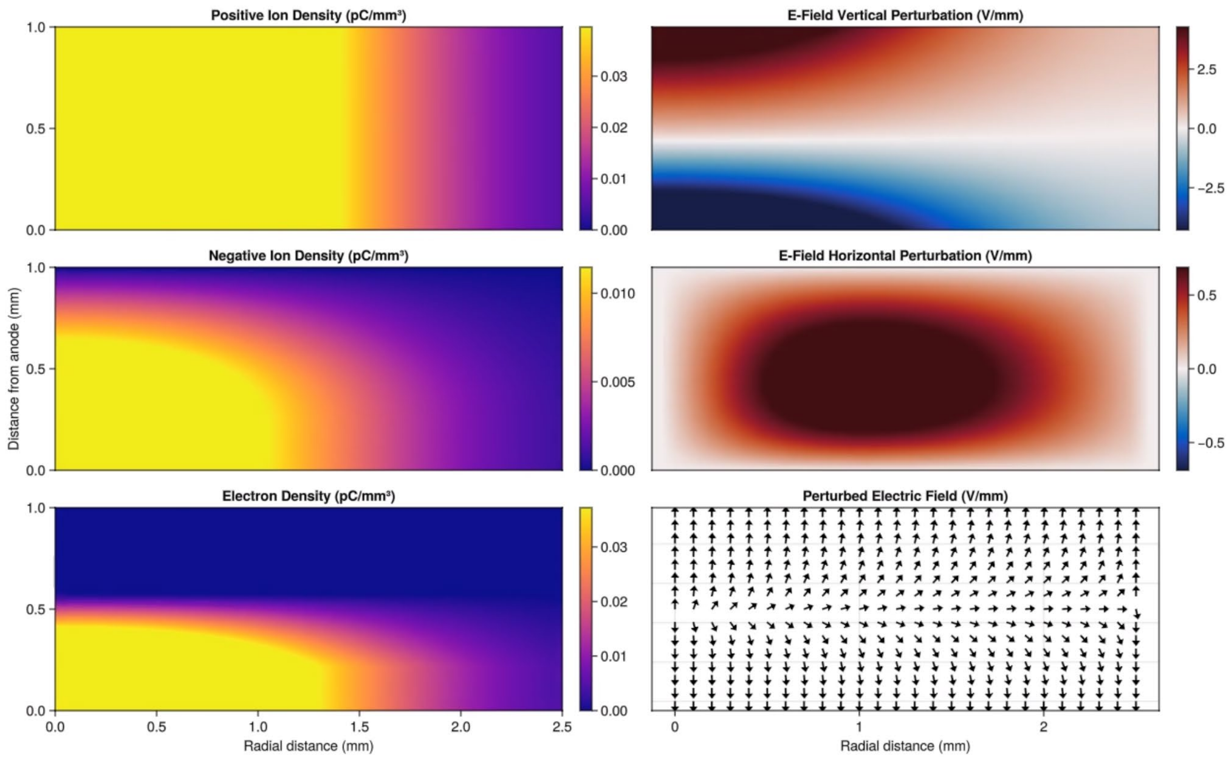

Figure 3. Radial slice of Parallel Plate Ion-Chamber, showing ion and electron charge densities and electric field perturbations.

Our optimized Julia solver enables simulation of a full axisymmetric system, capturing lateral charge dynamics that 1D approximations inherently miss. As shown in Figure 3, the solver captures complex multidimensional phenomena in which the intense Gaussian space charge screens the bias voltage and causes radial perturbations driving lateral ion deflection.

Conclusions:

We successfully characterized ion recombination effects for UHDR proton beams across various voltages and beam characteristics. The excellent agreement between measured data and physical model provides precise, temporally resolved insights into charge distribution and transport within an ionization chamber.

Accurately modeling these complex non-linear phenomena is essential for UHDR dosimetry and provides the foundational accuracy required for safe and effective FLASH radiotherapy. Furthermore, while parallel-plate chambers are useful for reference dosimetry, the flexibility of our FEA-FD solver will allow us to extend these corrections to cylindrical geometries, enabling the characterization of a wider range of clinical devices under UHDR conditions.

Relevance to CIRMS: This study introduces advanced tools for the measurement and simulation of UHDR radiation. Precise characterization of extremely ionizing beams is becoming increasingly important as FLASH clinical trials begin. Radiation Oncology relies on ADCL calibrations to ensure treatment safety and efficacy; this research directly supports the CIRMS mission by extending these standard principles to the spatiotemporal dynamics of FLASH ion-chamber based dosimetry. Finally, the first author is currently a PhD candidate aspiring to contribute to the field of medical physics in both research and clinical capacities.