NRC·CNRC

Transport under magnetic fields with the EGSnrc simulation toolkit

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National Research Council Canada

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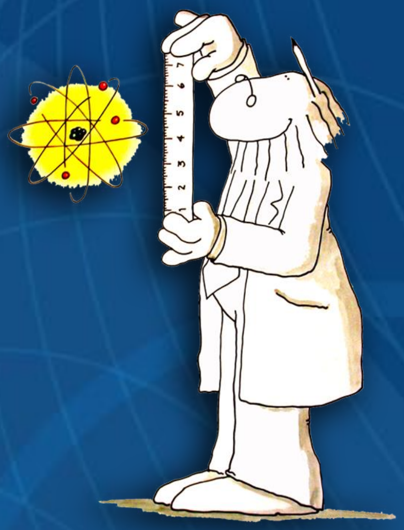
National Physics Laboratory (UK)



Can EGSnrc simulate ionization chamber response in the presence of magnetic fields accurately?

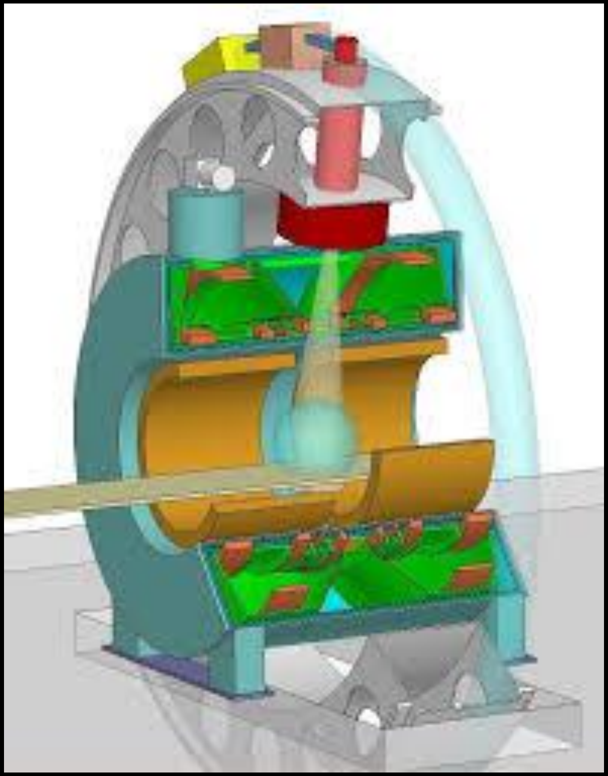
MRI

+

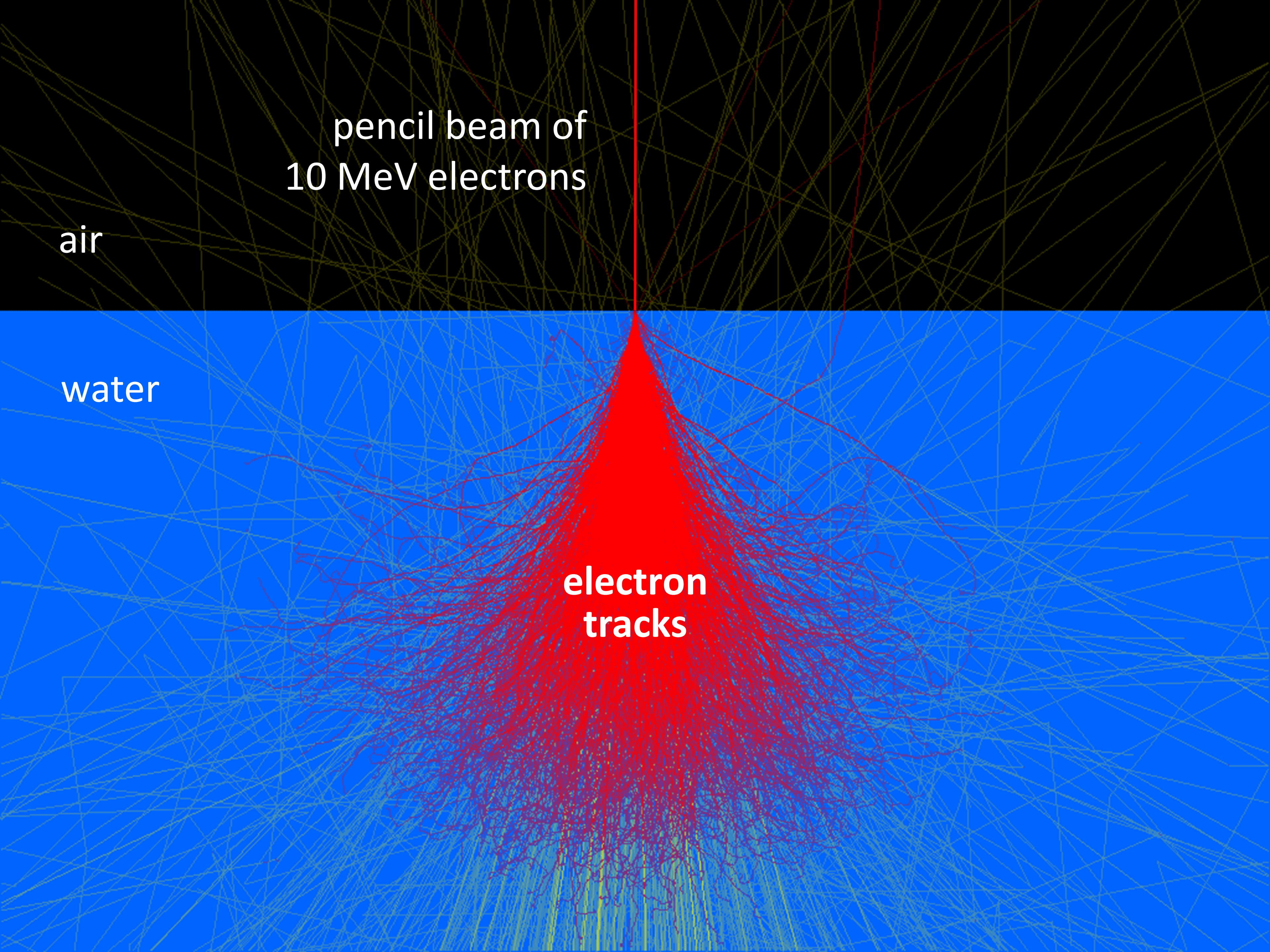
External RT

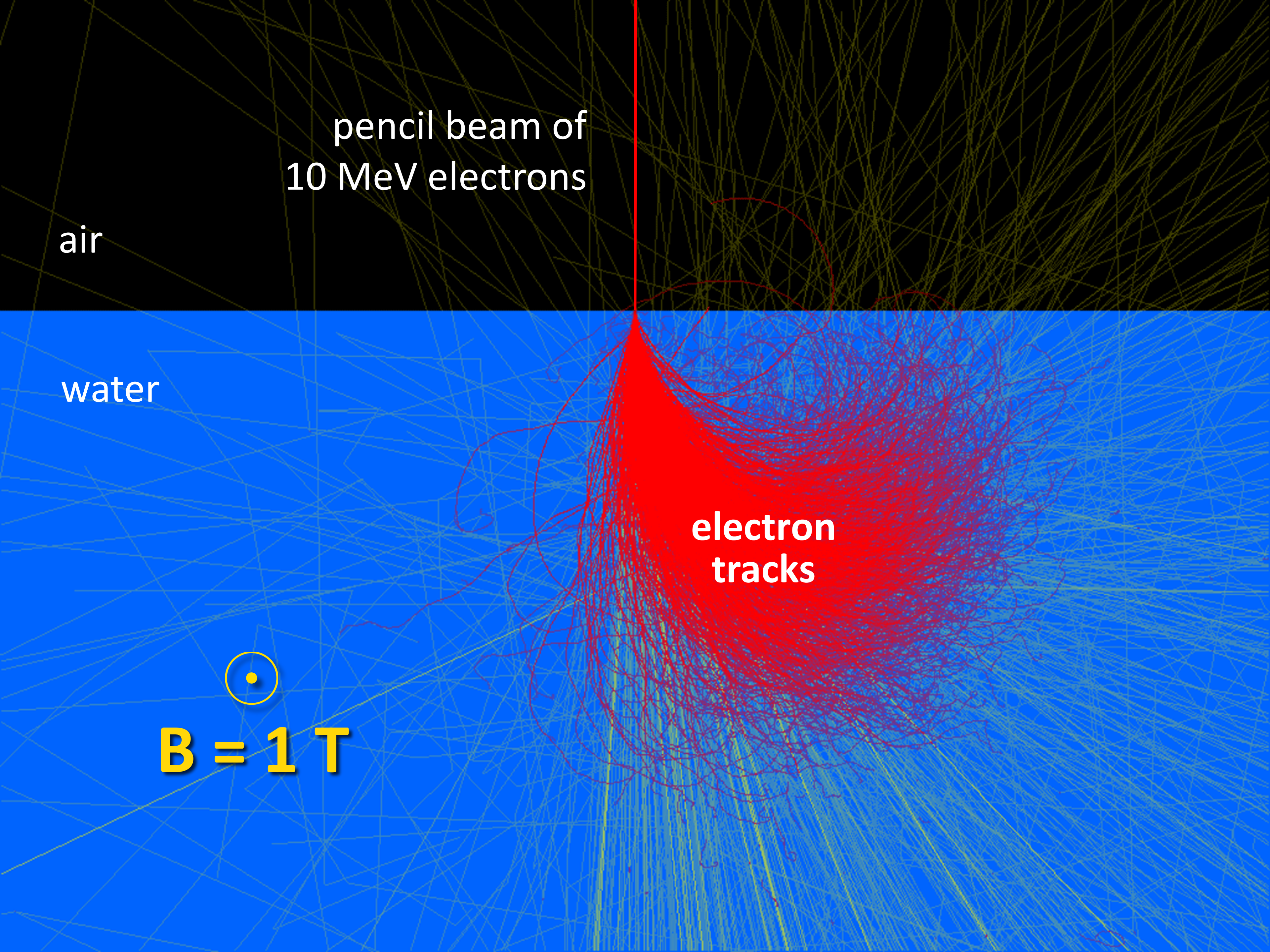
=

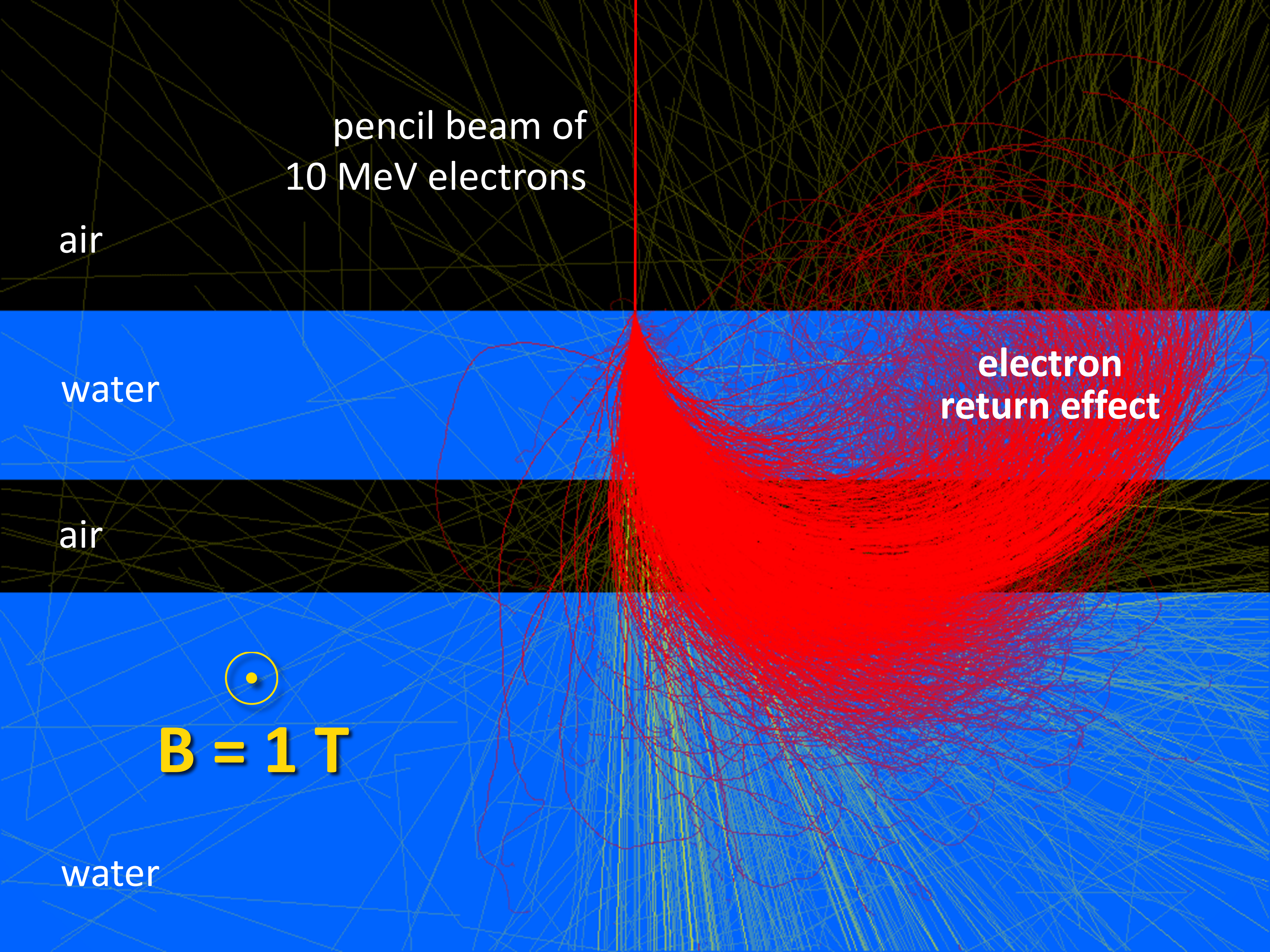
MRI-guided Radiation Therapy

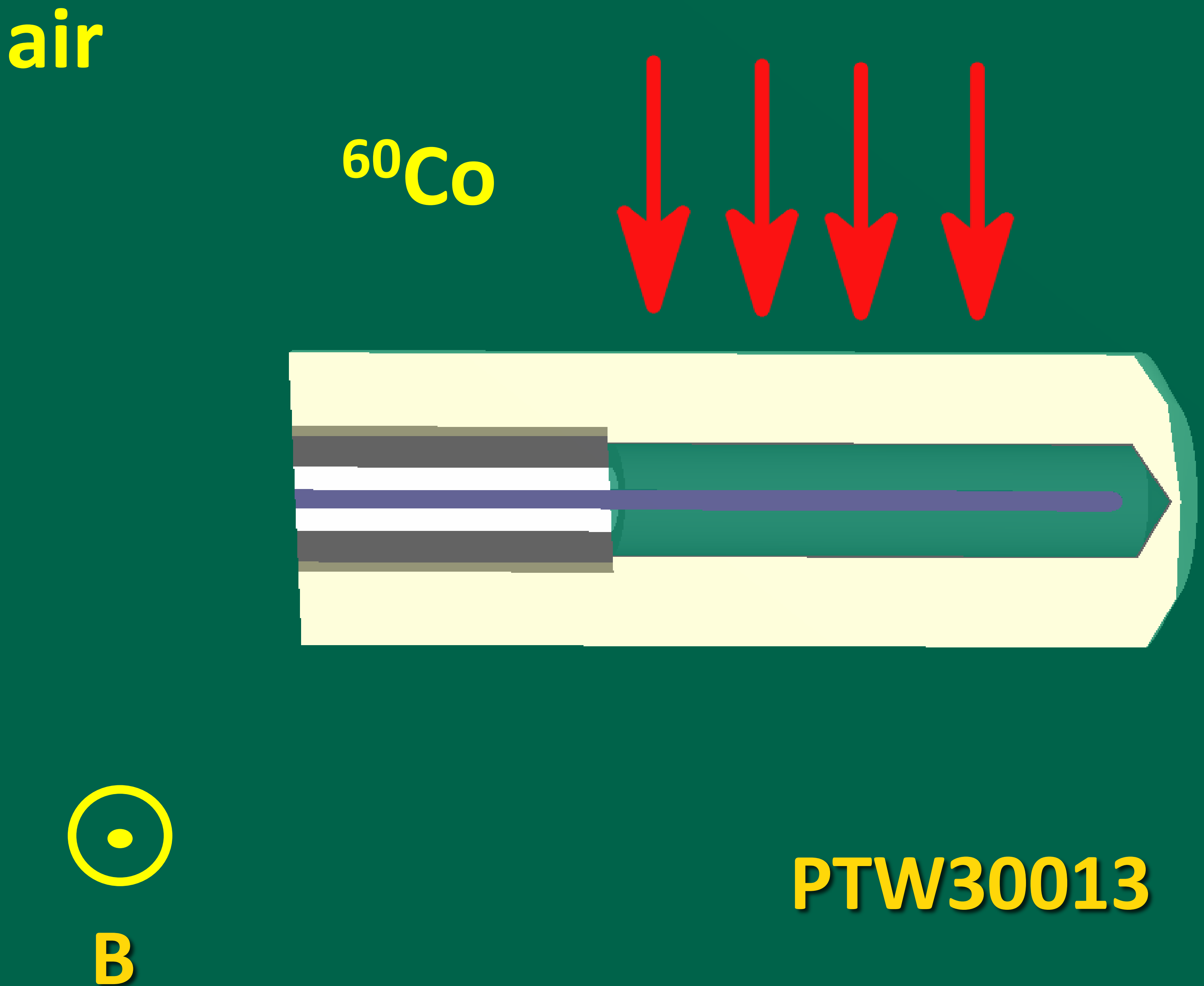


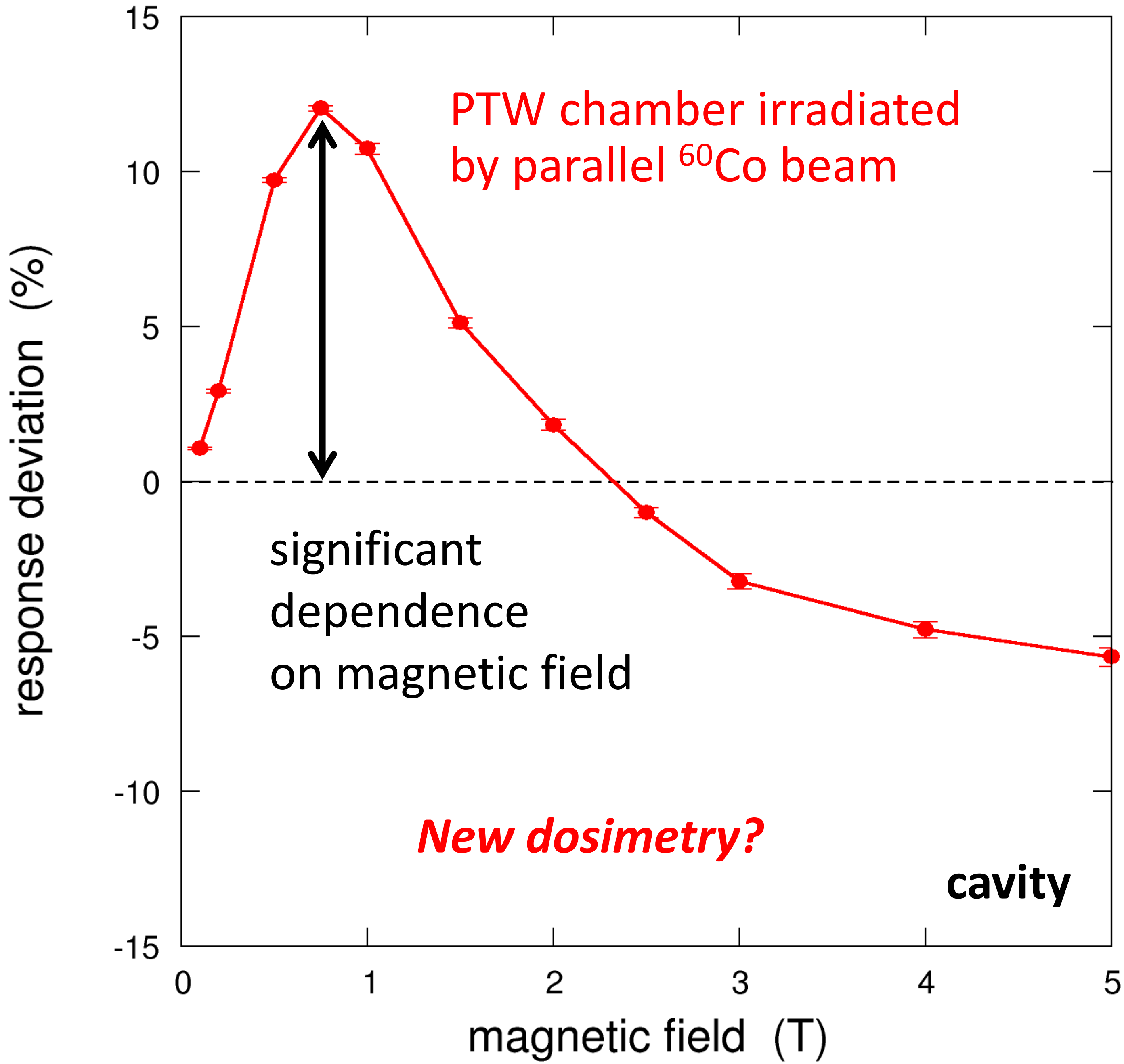
Magnetic field effect on dose distribution


pencil beam of
10 MeV electrons
air
water
electron
tracks

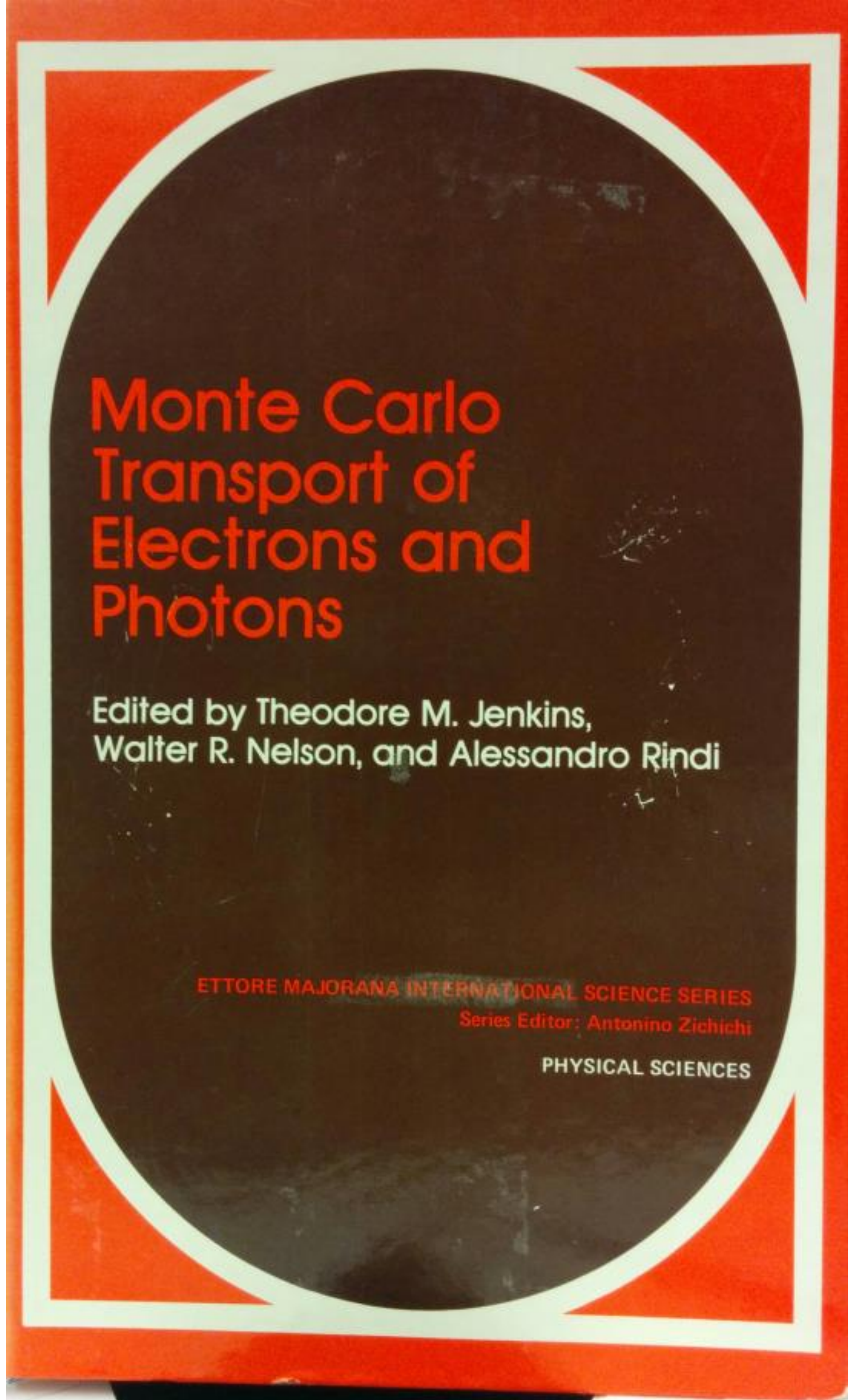

pencil beam of
10 MeV electrons
air
water
electron
tracks
B = 1 T


pencil beam of
10 MeV electrons
air
water
electron
return effect
air
B = 1 T
water


air
60Co
B
PTW30013


PTW chamber irradiated
by parallel ^{60}Co beam
significant
dependence
on magnetic field
New dosimetry?
cavity
response deviation (%)
magnetic field (T)
15
10
5
0
-5
-10
-15
0
1
2
3
4
5

Implementation



Orange Bible

19. Electron Transport in $\vec{E}$ and $\vec{B}$ Fields

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19.1 INTRODUCTION

In this chapter, we discuss the fundamentals of electron transport in static external electric and magnetic fields in vacuum and dense media. By "static" and "external" is meant that macroscopic $\vec{E}$ and/or $\vec{B}$ fields are set up in the region where the electron transport is taking place. For example, a high-energy particle detector may be placed in a constant magnetic field so that the momentum of charged particles may be analyzed. The external fields are considered to be static in the sense that they do not change with time during the course of the simulations. This is not a fundamental constraint, but is imposed for simplicity. The bulk of the discussion concerns the theoretical viability of performing electron transport in dense media in the presence of external fields. The trajectories of particles in this case can be quite complicated. The particles can be subjected to a myriad of forces — de-accelerations due to inelastic processes with orbital electrons and nuclei, elastic deflections due to attraction or repulsion in the nuclear electric field, accelerations or de-accelerations by the external electric field, and deflections by the external electric and magnetic fields.

In comparison to the effects of the internal processes of multiple scattering and inelastic collisions, the effect of the external fields can be quite dramatic. Electric field strengths can be as high as 2 MV/(g/cm^2). The rate of a charged particle's change in energy due to this field can be equal in magnitude to the rate of energy loss of high-energy electrons in matter. We wish to establish a method, even if it is a "brute force" one, that will allow us to do charged-particle transport under these circumstances. We do not wish to treat the effects of the external fields as perturbations on the field-free transport in media. Yet, we don't wish to discard all the theoretical work that has been achieved in field-free transport. Rather, we shall retain what we know about inelastic energy-loss mechanisms and multiple scattering, and attempt to include the effect of the external fields, albeit in a simple-minded fashion.

We commence the chapter with a "review" discussion of charged-particle transport in external fields. We set up the equations and then solve them in vacuum. The vacuum solutions will play a role in the benchmarking of the differential equations as modelled in the Monte Carlo code. We then prove formally under which circumstances the vacuum transport equations can be "tacked on" to the field-free transport with little error. In

Equation of motion

The equation of motion in the force formulation for transport in a medium under the effect of an EM field can be written as

$$\vec{v} = \vec{v}_0 + \frac{1}{m_0\gamma(E)}\int_0^t dt'\{\underbrace{\vec{\boldsymbol{F}}_{\boldsymbol{el}}\big(E(t')\big) + \vec{\boldsymbol{F}}_{\boldsymbol{in}}\big(E(t')\big)}_{\text{stochastic}} + \underbrace{\vec{\boldsymbol{F}}_{\boldsymbol{em}}\big(\vec{x}(t'), E(t'), \hat{u}(t')\big)}_{\text{deterministic}}\}$$

stochastic **deterministic**

Bielajew's implementation

Under the assumption of **very small steps** such that:

- Field does not changes significantly
- Energy loss negligible
- Negligible angular deflection

the equation of motion becomes to first order:

$$\vec{v} = \vec{v}_0 + \frac{t}{m_0 \gamma(E_0)} \left\{ \overrightarrow{\boldsymbol{F}}_{\boldsymbol{el}}(E_0) + \overrightarrow{\boldsymbol{F}}_{\boldsymbol{in}}(E_0) + \overrightarrow{\boldsymbol{F}}_{\boldsymbol{em}}(\vec{x}_0, E_0, \hat{u}_0) \right\}$$

Bielajew's implementation

Under the assumption of **very small steps** such that:

- Field does not changes significantly
- Energy loss negligible
- Negligible angular deflection

the equation of motion becomes to first order:

$$\vec{v} = \vec{v}_0 + \Delta\vec{v}_{MC} + \frac{t}{m_0\gamma(E_0)}\{\overrightarrow{\boldsymbol{F}}_{\boldsymbol{em}}(\vec{x}_0, E_0, \hat{u}_0)\}$$

Interactions with medium and external field treated independently!

Bielajew's implementation

Expressing the time t as a function of the total path length Δs to first order gives

$$\Delta\vec{x} = \hat{u}_0 \Delta s + \frac{\Delta s}{2} \Delta\hat{u}$$

Neglecting lateral deflection Δs/2 one gets for the position change

$$\Delta\vec{x} = \hat{u}_0 \Delta s$$

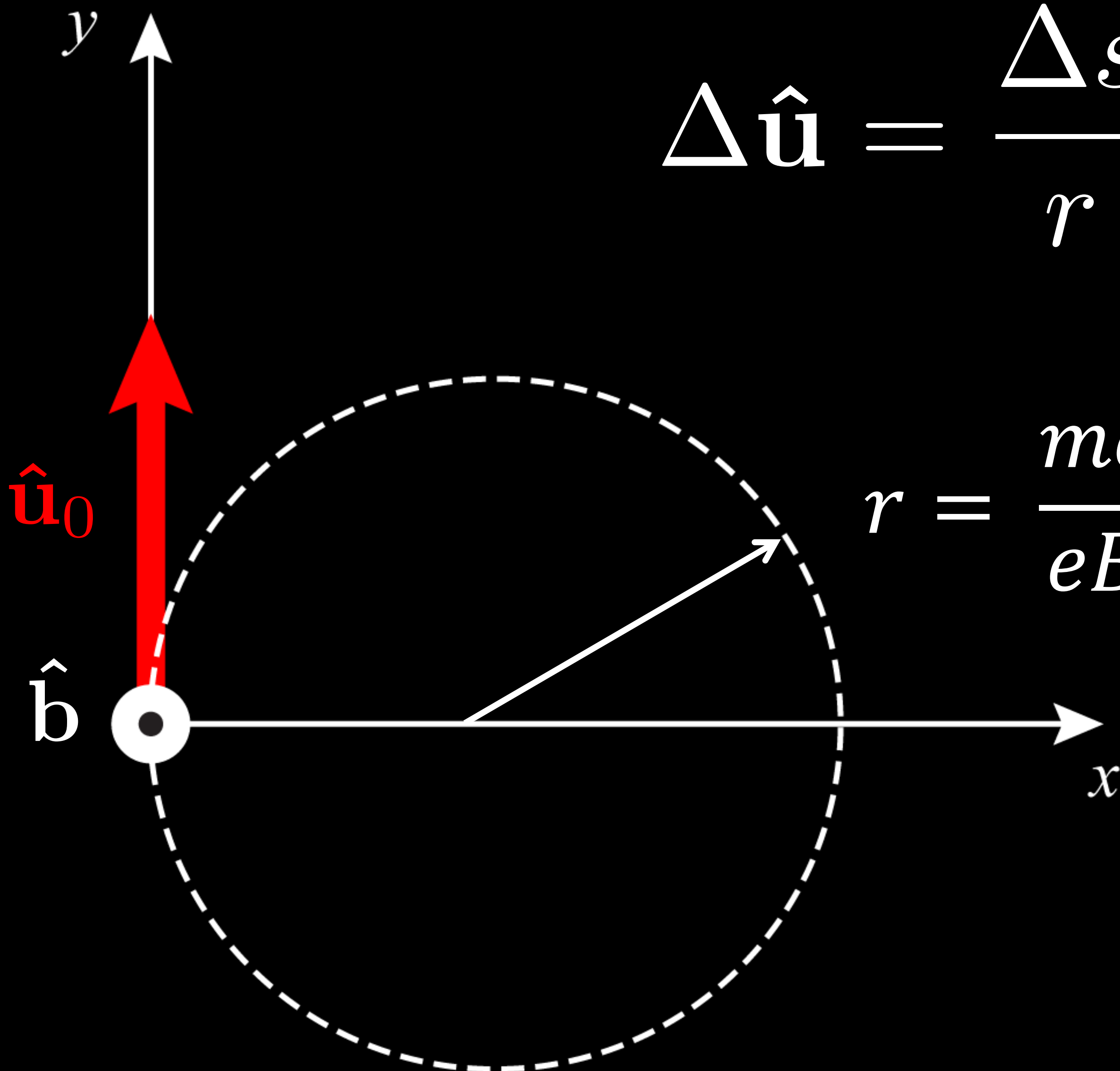
MC step

the change in the particle's direction is

$$\Delta\hat{u} = \Delta\hat{u}_{MC} + \Delta\hat{u}_{em}$$

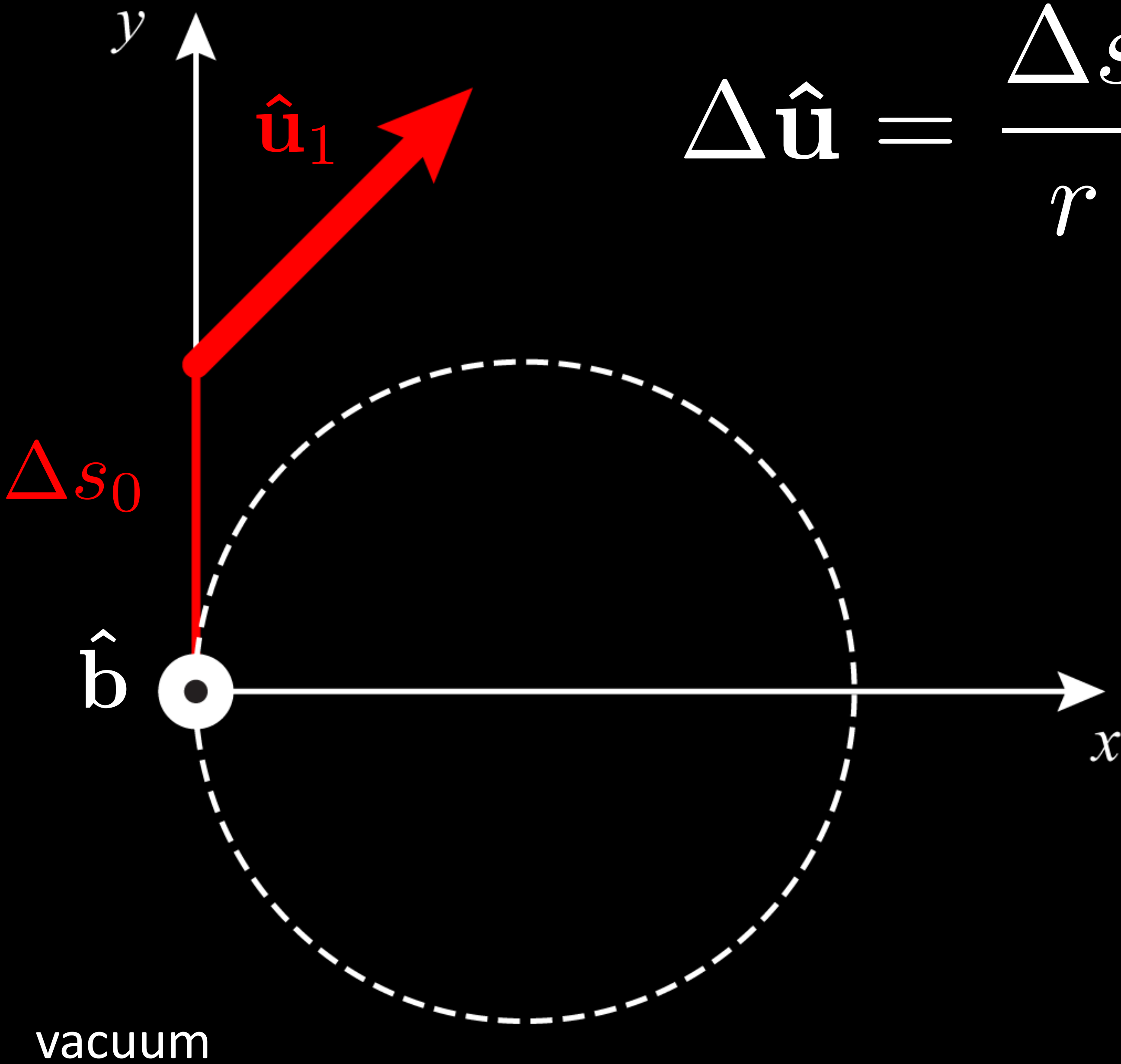
$$\Delta \hat{\mathbf{u}} = \frac{\Delta s}{r}(\hat{\mathbf{u}} \times \hat{\mathbf{b}})$$

$$r = \frac{mc}{eB}\sqrt{\gamma^2 - 1}$$

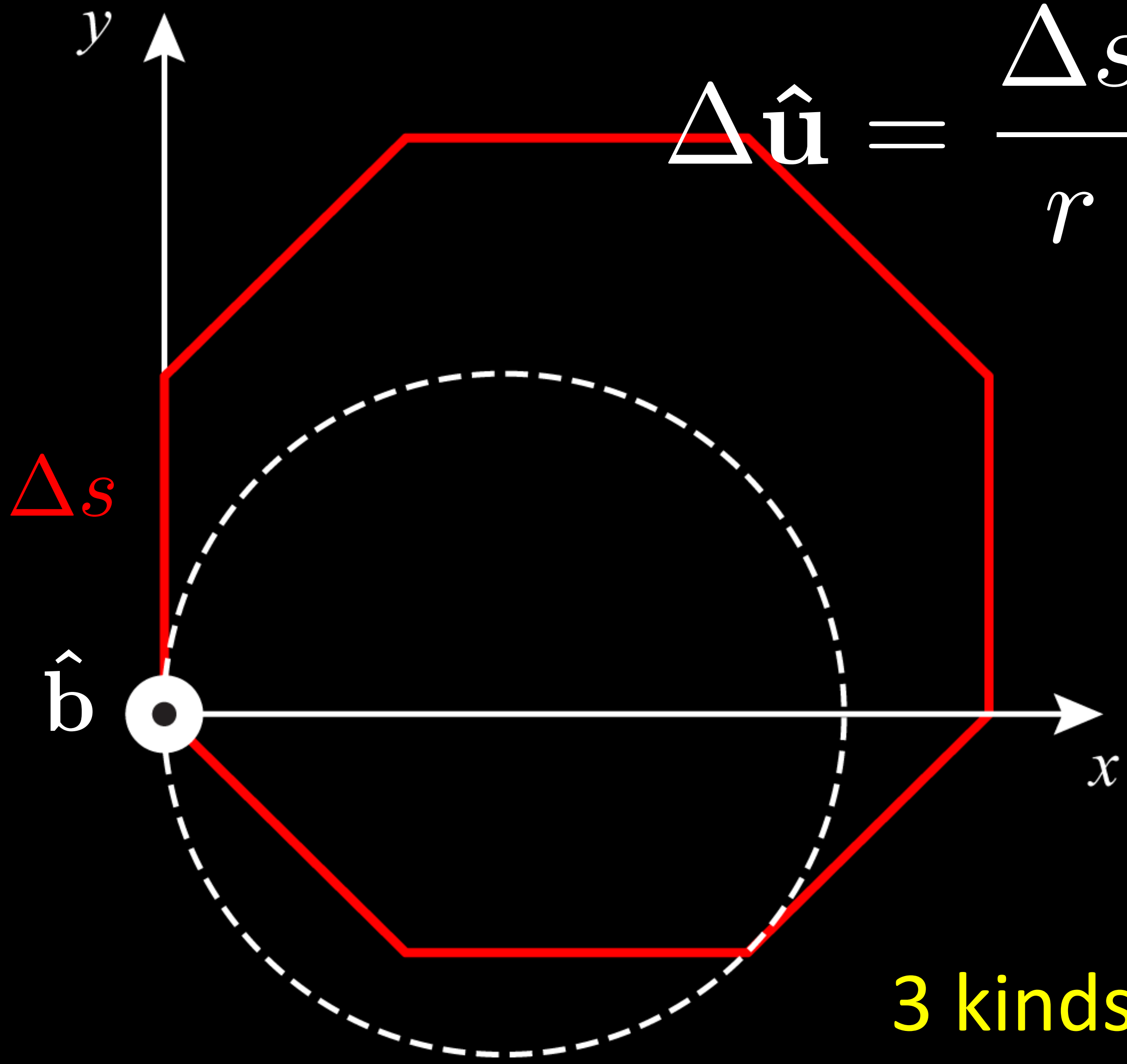


vacuum

$$\Delta\hat{\mathbf{u}} = \frac{\Delta s}{r}(\hat{\mathbf{u}} \times \hat{\mathbf{b}})$$



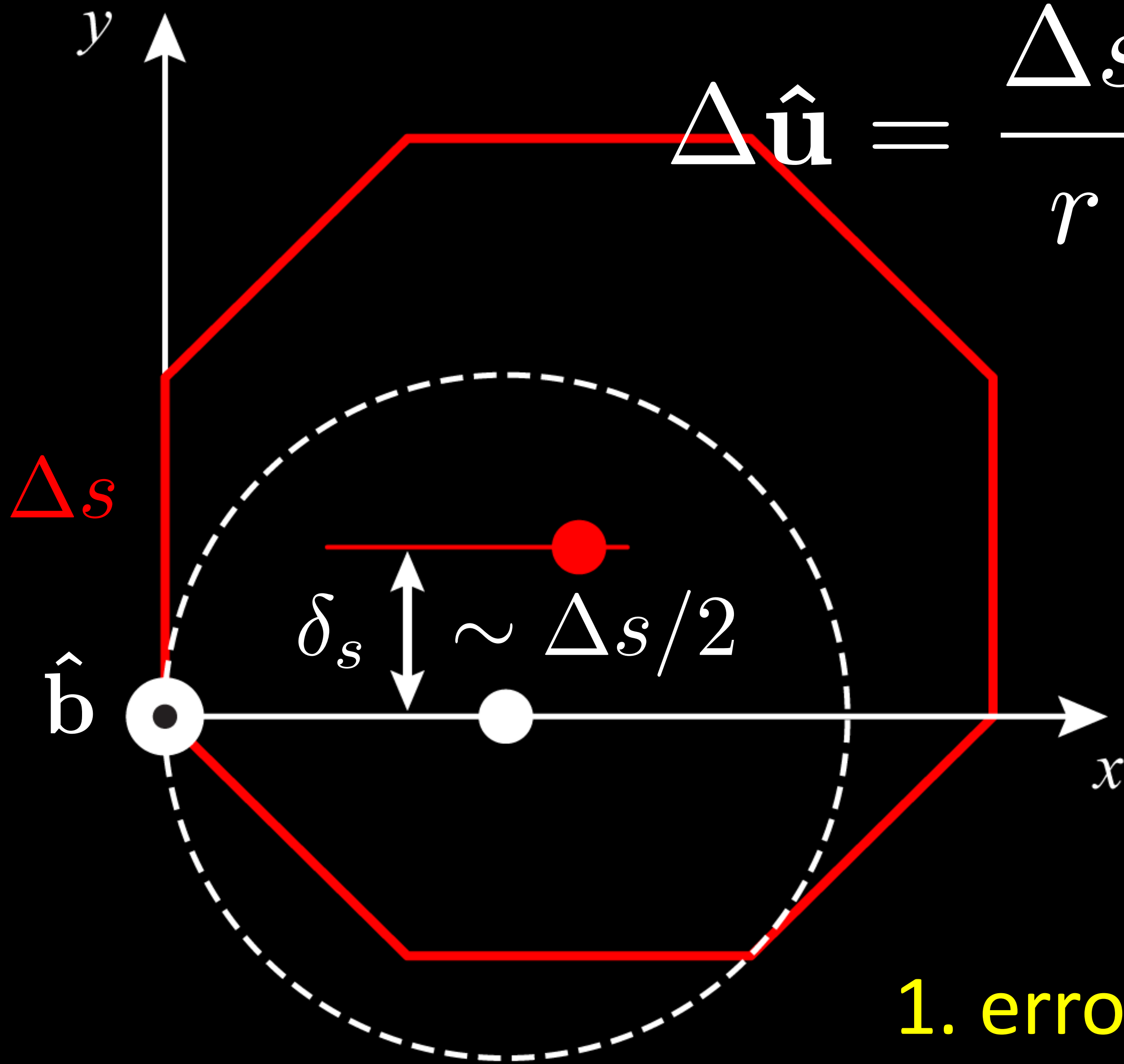
$$\Delta\hat{\mathbf{u}} = \frac{\Delta s}{r}(\hat{\mathbf{u}} \times \hat{\mathbf{b}})$$



3 kinds of errors

vacuum

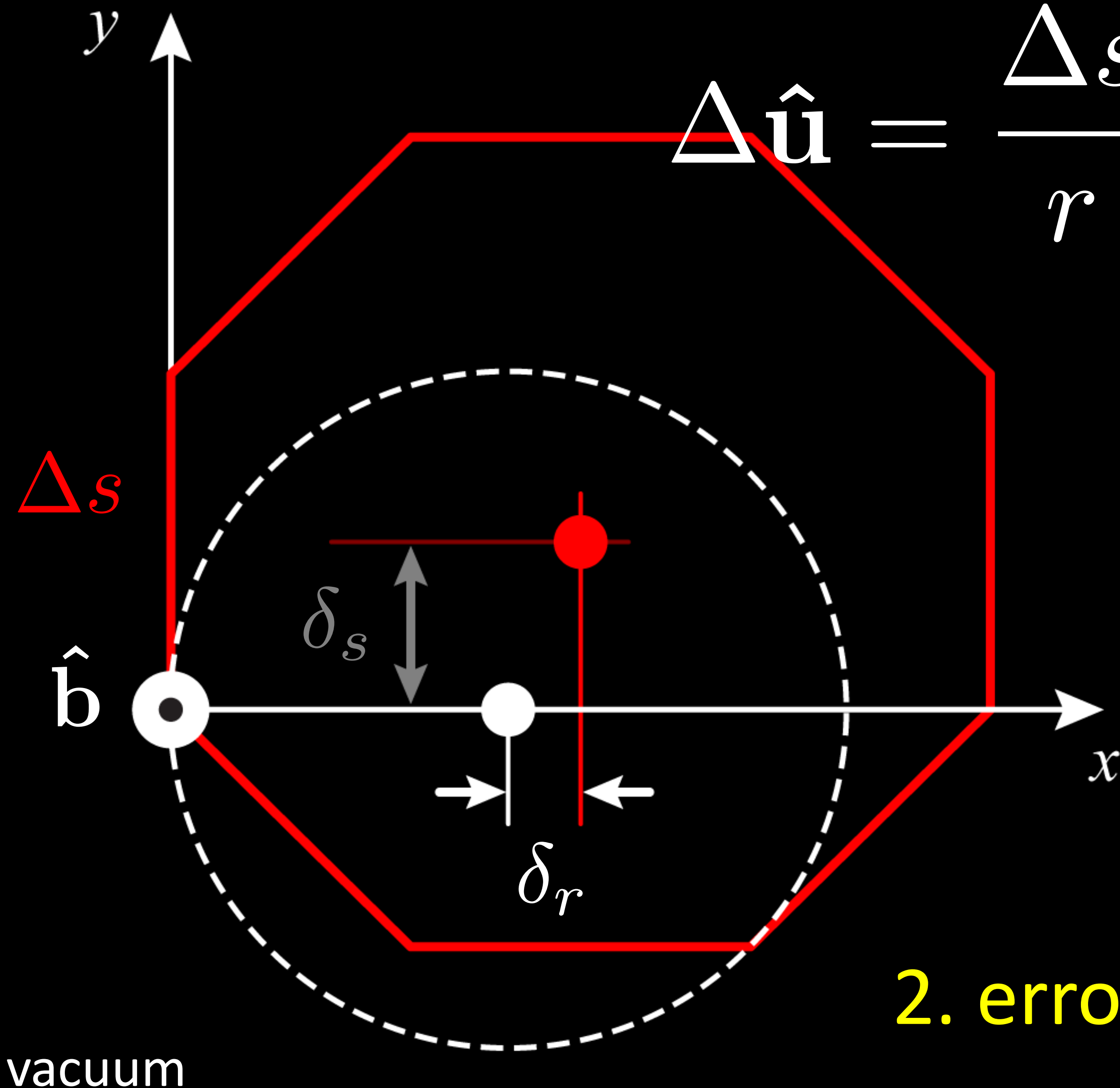
$$\Delta\hat{\mathbf{u}} = \frac{\Delta s}{r}(\hat{\mathbf{u}} \times \hat{\mathbf{b}})$$



1. error in position

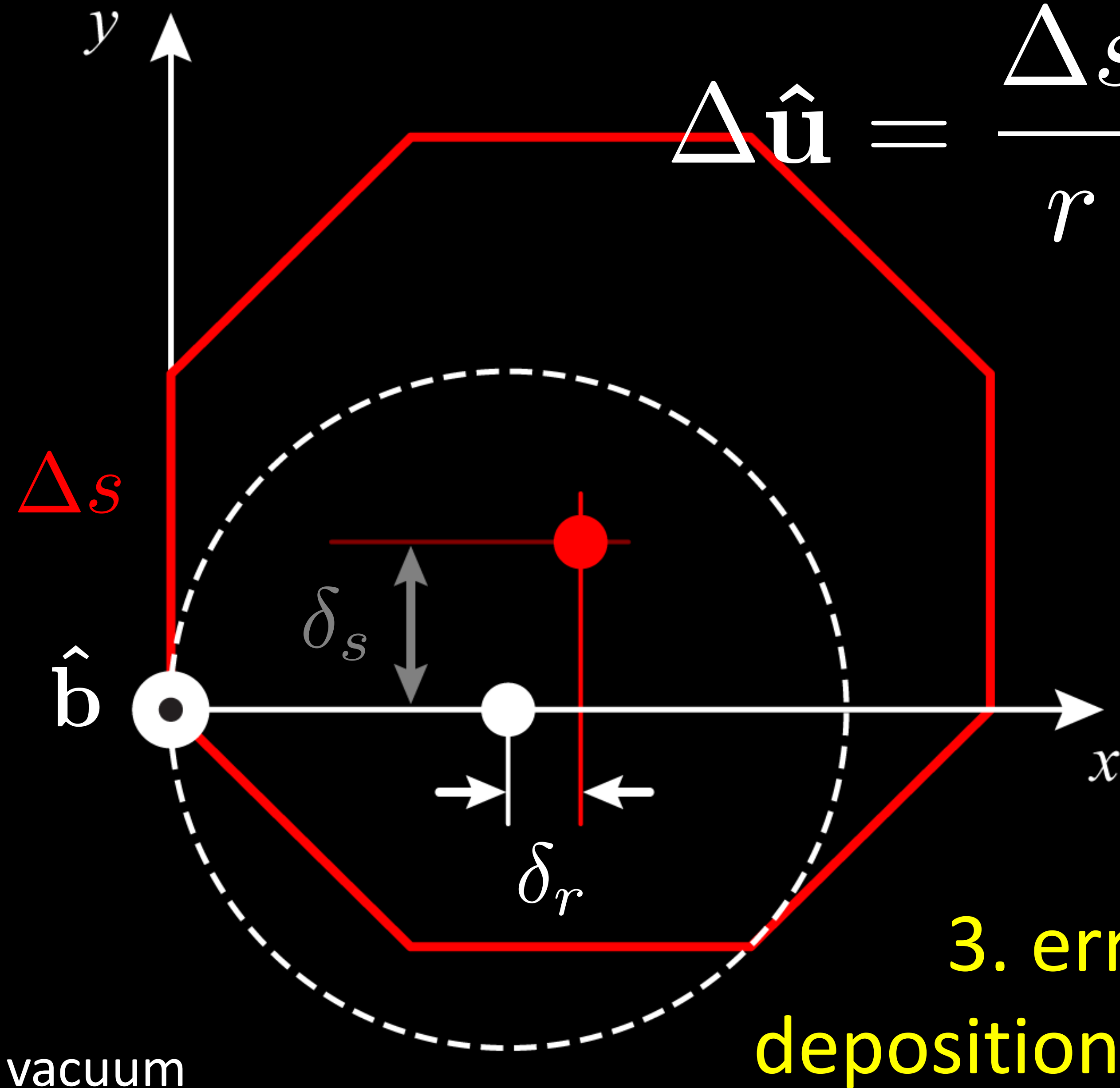
vacuum

$$\Delta\hat{\mathbf{u}} = \frac{\Delta s}{r}(\hat{\mathbf{u}} \times \hat{\mathbf{b}})$$



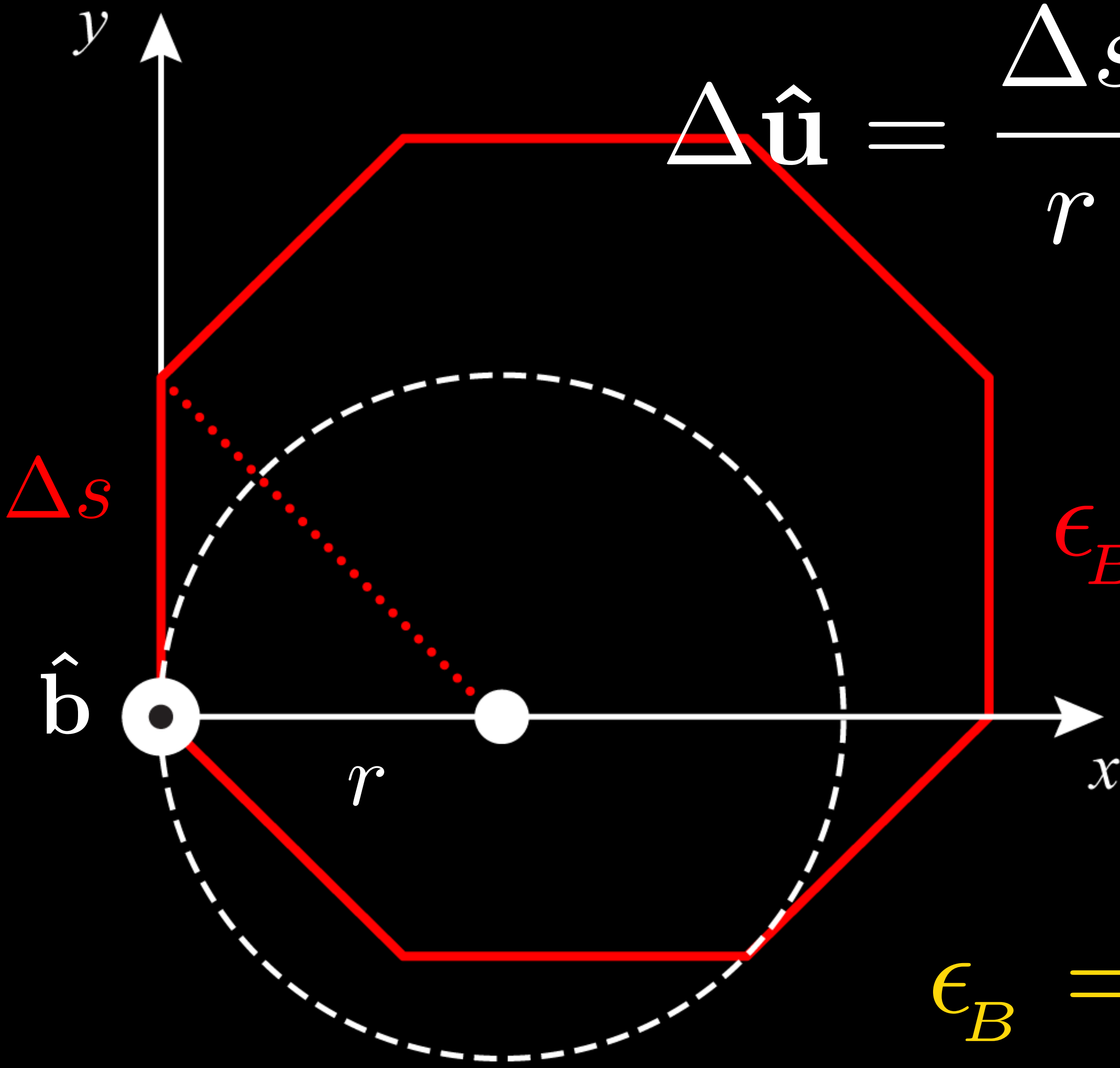
2. error in radius

$$\Delta\hat{\mathbf{u}} = \frac{\Delta s}{r}(\hat{\mathbf{u}} \times \hat{\mathbf{b}})$$



3. error in energy deposition (in medium)

$$\Delta \hat{\mathbf{u}} = \frac{\Delta s}{r}(\hat{\mathbf{u}} \times \hat{\mathbf{b}})$$

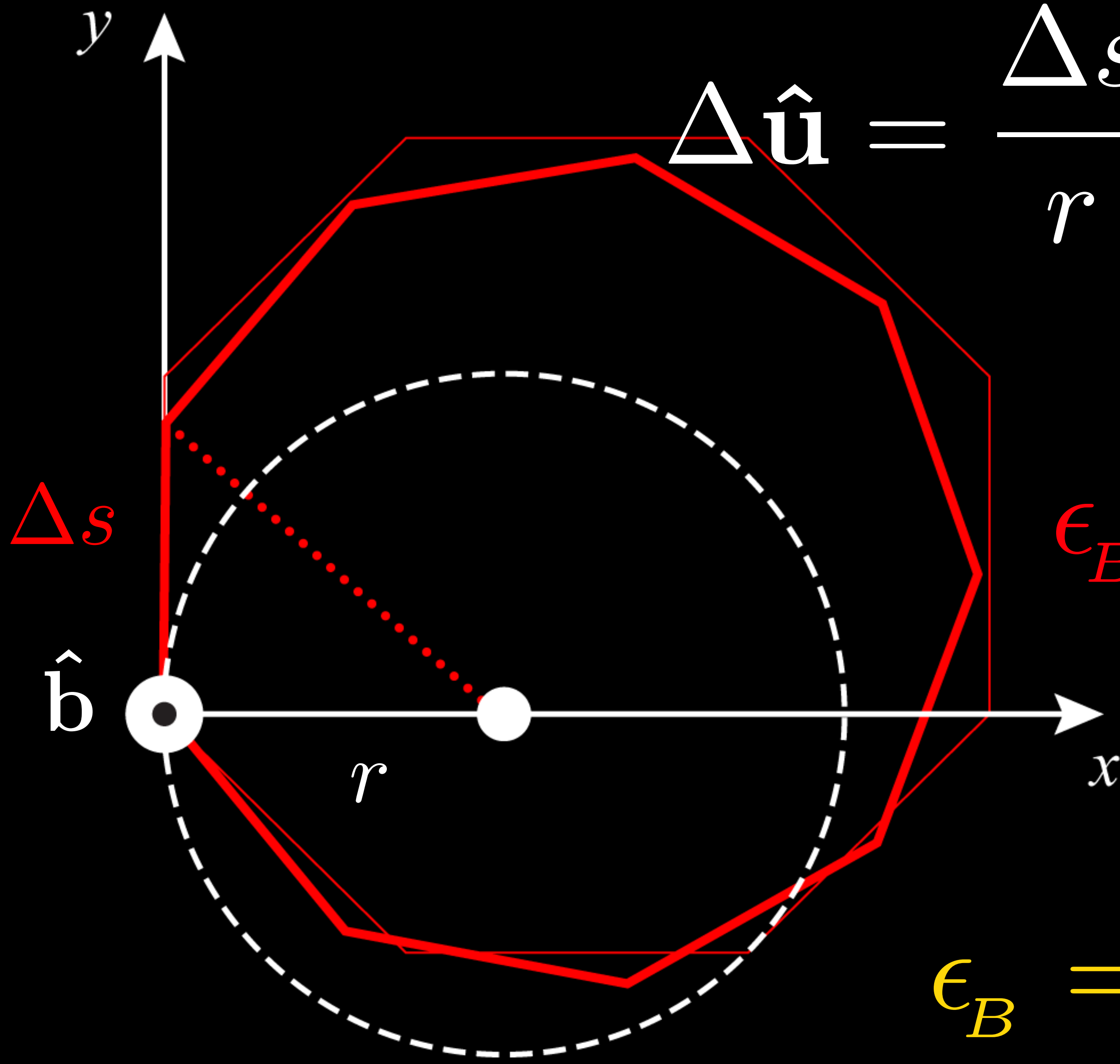


$$\epsilon_B = 1$$

$$\epsilon_B = \frac{\Delta s_{\mathrm{max}}}{r}$$

vacuum

$$\Delta \hat{\mathbf{u}} = \frac{\Delta s}{r}(\hat{\mathbf{u}} \times \hat{\mathbf{b}})$$

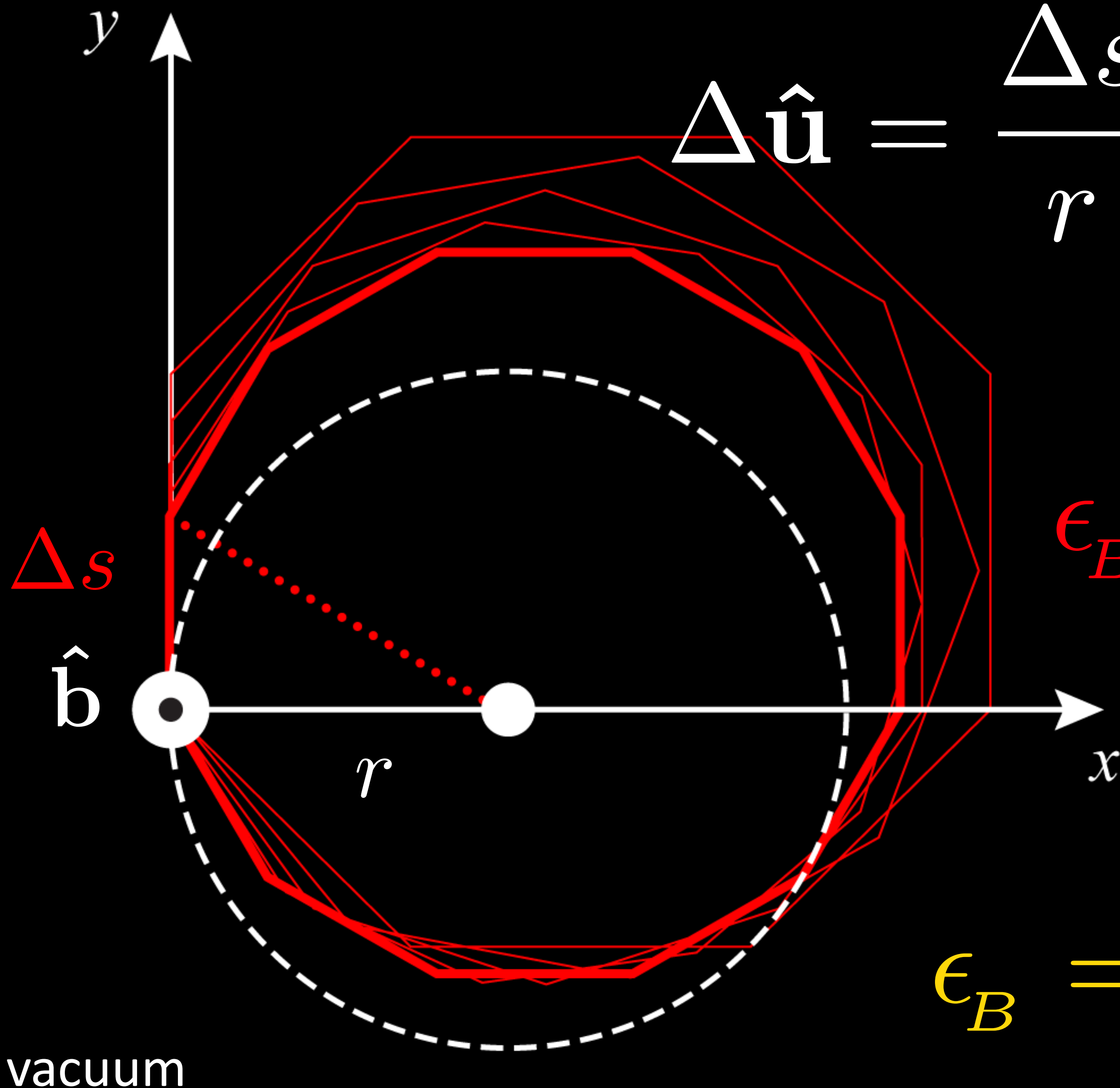


$$\epsilon_B = 0.84$$

$$\epsilon_B = \frac{\Delta s_{\mathrm{max}}}{r}$$

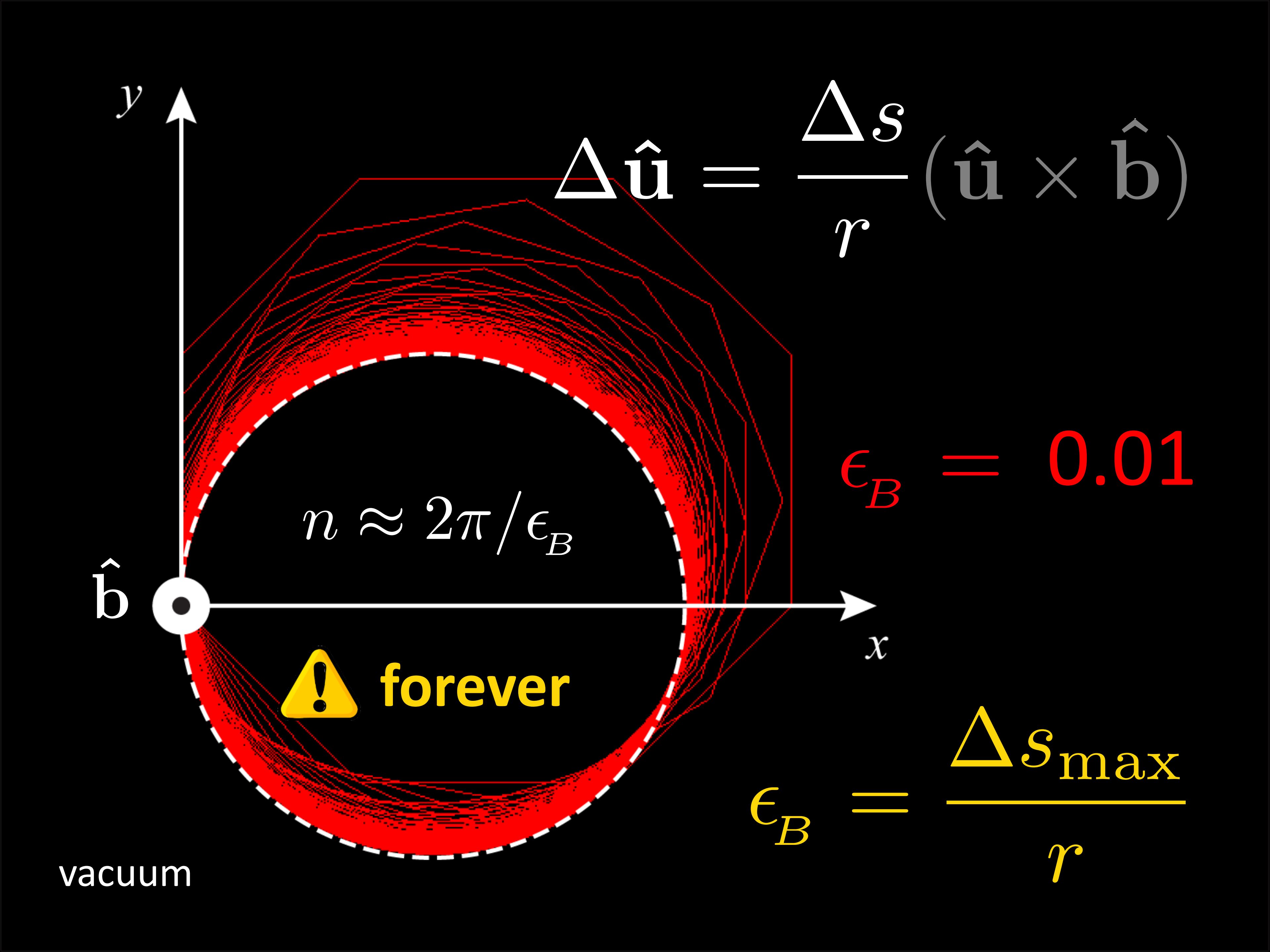
vacuum

$$\Delta\hat{\mathbf{u}} = \frac{\Delta s}{r}(\hat{\mathbf{u}} \times \hat{\mathbf{b}})$$



$$\epsilon_B = 0.58$$

$$\epsilon_B = \frac{\Delta s_{\max}}{r}$$


$\Delta\hat{\mathbf{u}} = \frac{\Delta s}{r}(\hat{\mathbf{u}} \times \hat{\mathbf{b}})$
y
x
$\hat{\mathrm{b}}$
$n \approx 2\pi/\epsilon_B$
$\epsilon_B =$ 0.01
forever
$\epsilon_B = \frac{\Delta s_{\max}}{r}$
vacuum

Fano test

Note on the Bragg-Gray Cavity Principle for Measuring Energy Dissipation

U. FANO

National Bureau of Standards, Washington, D. C.

that, whereas the compression increases the number of secondary electrons generated per unit volume, it reduces the "range" of each electron by an equal factor.[4] In fact, the argument rests on a much more detailed theorem which has been often implied but perhaps never stated in full.

Theorem: In a medium of given composition exposed to a uniform flux of primary radiation (such as X-rays or neutrons) the flux of secondary radiation is also uniform and *independent of the density* of the medium as well as of the density variations from point to point.

U. Fano (*1954*) Note on the Bragg-Gray Cavity Principle for Measuring Energy Dissipation. Radiation Research: June 1954, Vol. 1, No. 3, pp. 237-240.

RADIATION RESEARCH **63**, 191–199 (1975)

Some Comments on Fano's Theorem[1]

L. V. SPENCER

National Bureau of Standards, Washington, D. C. 20234

REVISED STATEMENT OF THE THEOREM, AND FANO'S PROOF

In what follows, we use "fluence" rather than the more accurate but longer term "differential fluence," in a manner very similar to Fano's use of "flux."

Let us begin with a modified statement of the *Theorem*:

In an unbounded medium of uniform composition containing a source of electrons which is everywhere proportional to the local density, the fluence of electrons is uniform and is independent of density variations.

Fano theorem provides a rigorous test

- Uniform electron source per unit mass N_0/m_T
- Medium of uniform composition but varying density

$$D = N_0/m_T \cdot \langle E \rangle$$

where $\langle E \rangle$ is the average energy emitted

Fano theorem provides a rigorous test

If the source emits electrons of energy E_0:

$$D/N_0 = E_0/m_T$$

For a MC simulation fulfilling Fano conditions, the dose per particle in ***any*** region ***i*** is expected to be:

$$D_i/N_0 = E_0/m_T$$

Use this to **verify the accuracy of the electron transport algorithm!**

Is Fano's theorem valid in the presence of magnetic fields ?

IOP Publishing | Institute of Physics and Engineering in Medicine   Physics in Medicine & Biology

Phys. Med. Biol. **60** (2015) 4963–4971   doi:10.1088/0031-9155/60/13/4963

Lorentz force correction to the Boltzmann radiation transport equation and its implications for Monte Carlo algorithms

Hugo Bouchard[1] **and Alex Bielajew**[2]

[1] Acoustics and Ionising Radiation Team, National Physical Laboratory, Hampton Road, Teddington TW11 0LW, UK
[2] Department of Nuclear Engineering and Radiological Sciences, The University of Michigan, Ann Arbor, MI 48109, USA

"Fano's theorem does not hold in the presence of static and constant external EM fields. ***This has the unfortunate consequence of invalidating the Fano cavity test*** *…"*

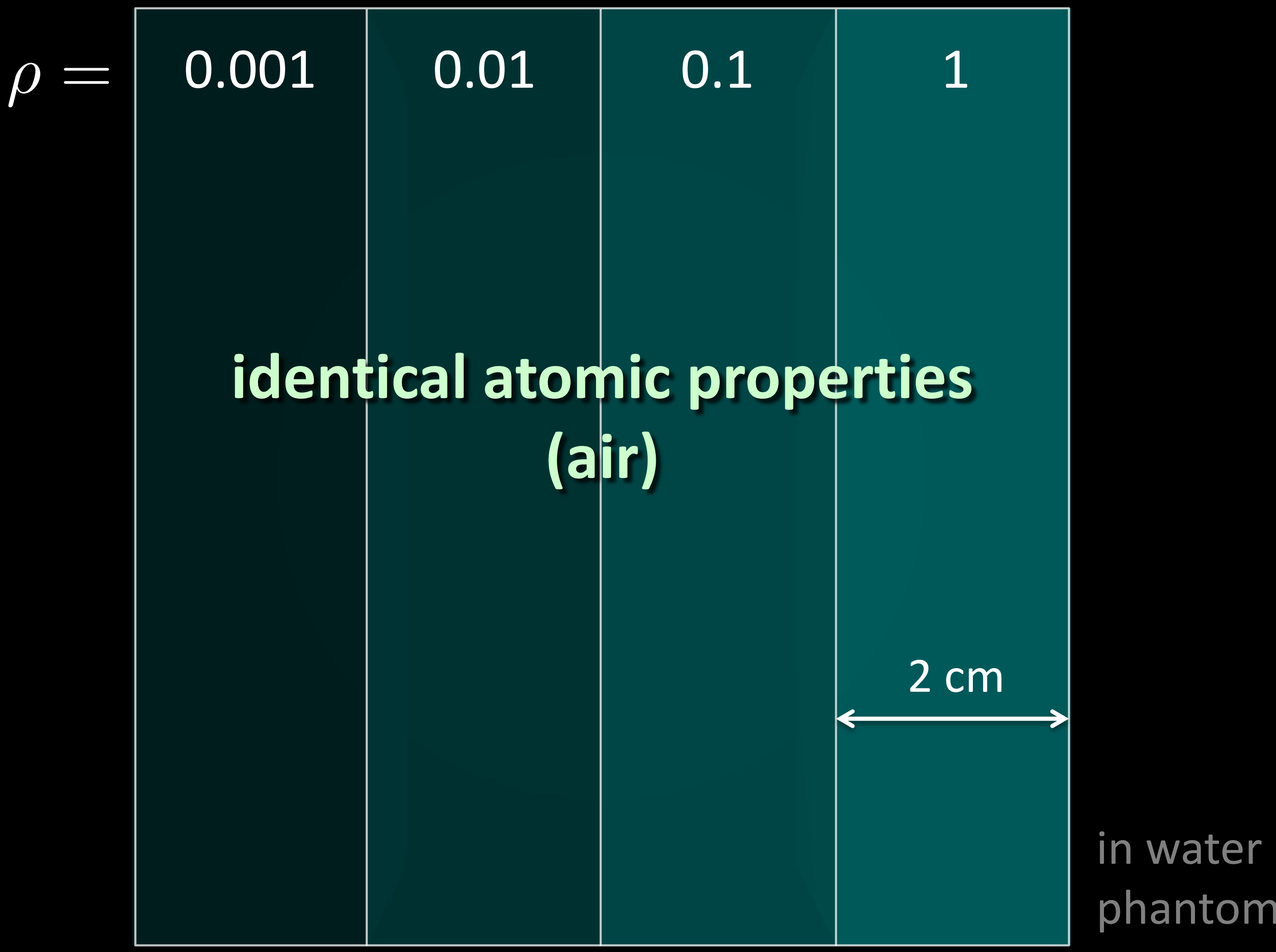
IOP Publishing | Institute of Physics and Engineering in Medicine Physics in Medicine & Biology

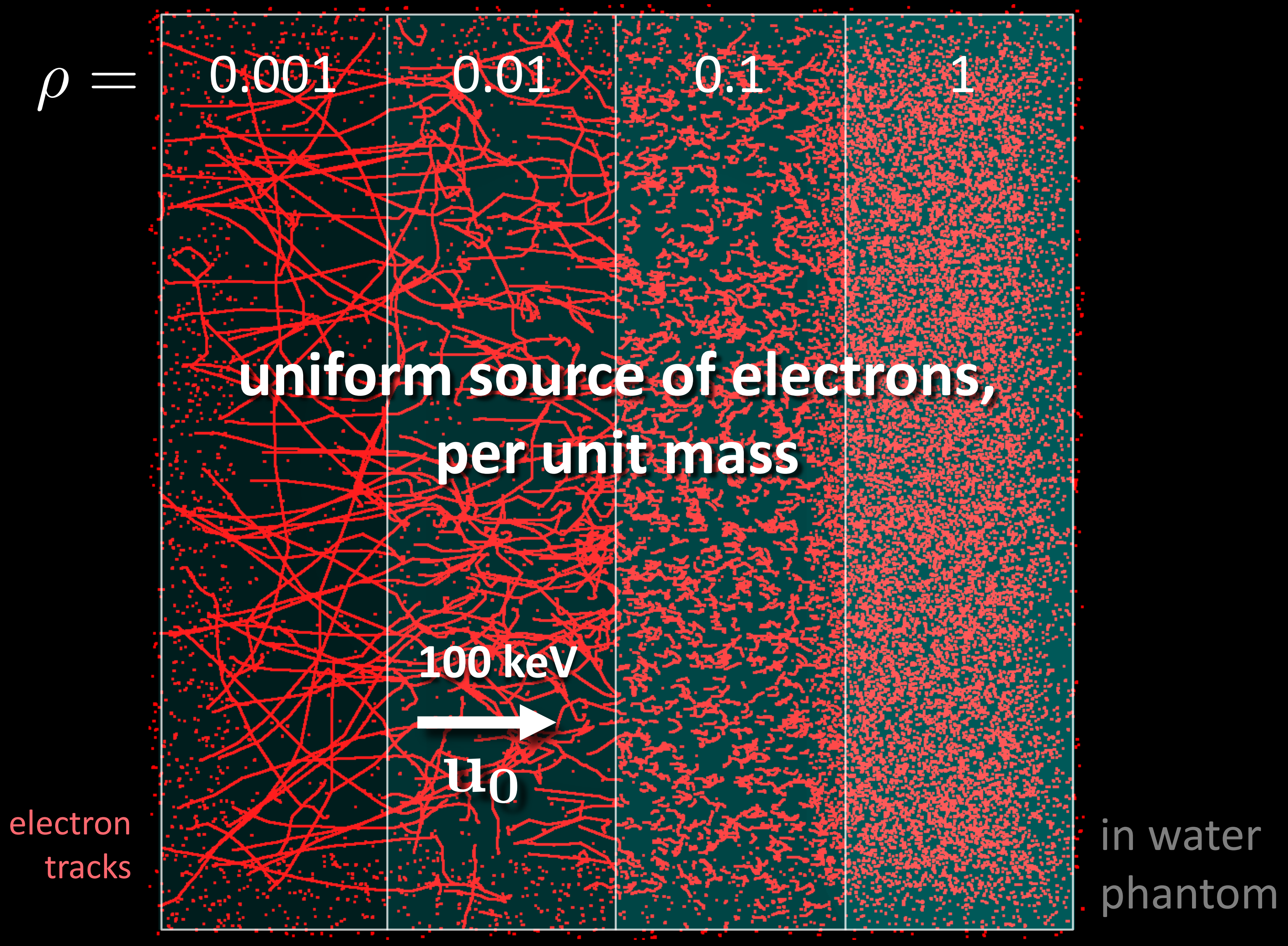
Phys. Med. Biol. 60 (2015) 6639–6654 doi:10.1088/0031-9155/60/17/6639

Reference dosimetry in the presence of magnetic fields: conditions to validate Monte Carlo simulations

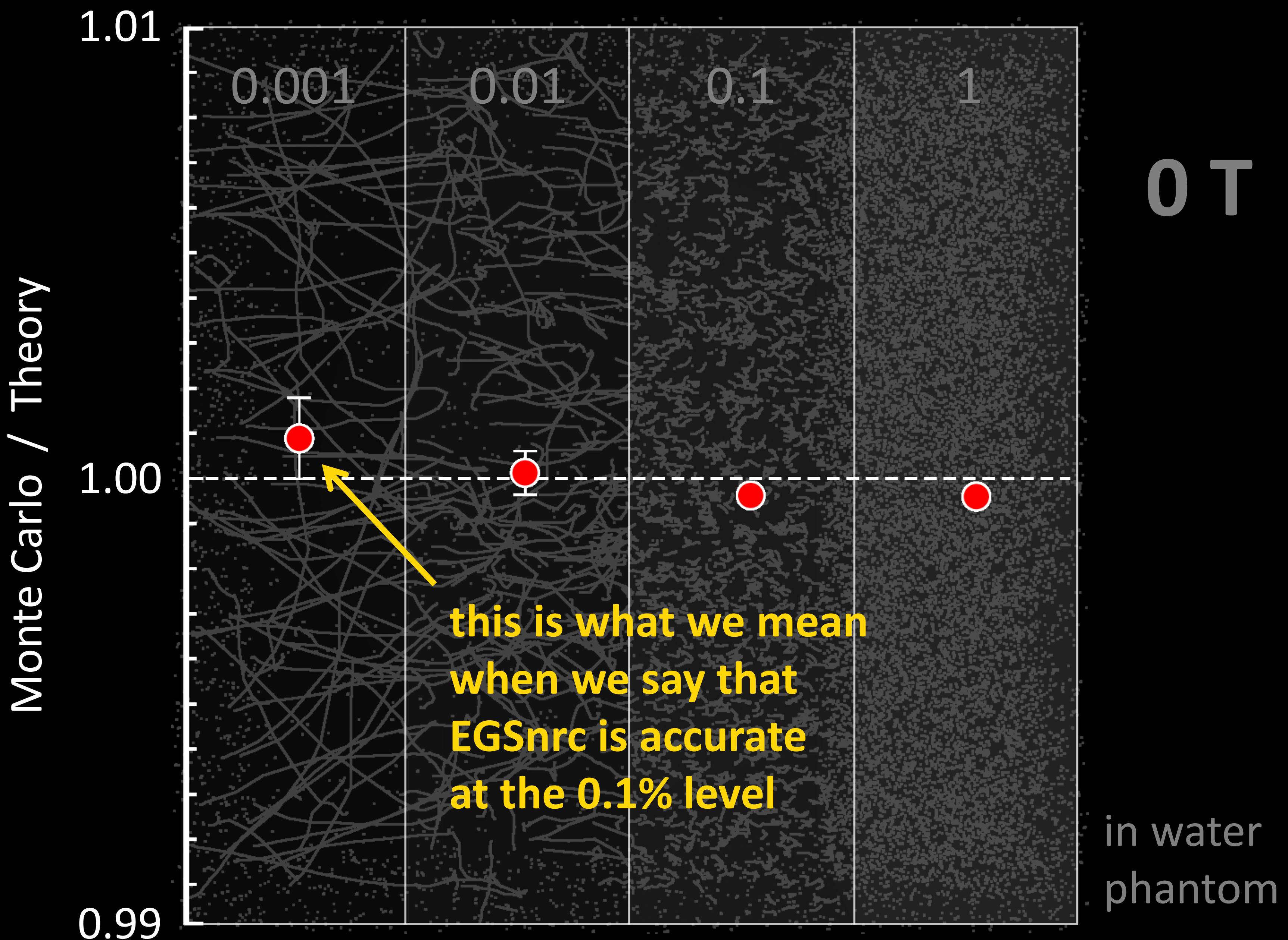
Hugo Bouchard[1], Jacco de Pooter[2], Alex Bielajew[3] and Simon Duane[1]

1. **Isotropic uniform source per unit mass**
2. **Magnetic field B scales with mass density**


$\rho =$
0.001
0.01
0.1
1
identical atomic properties
(air)
2 cm
in water
phantom

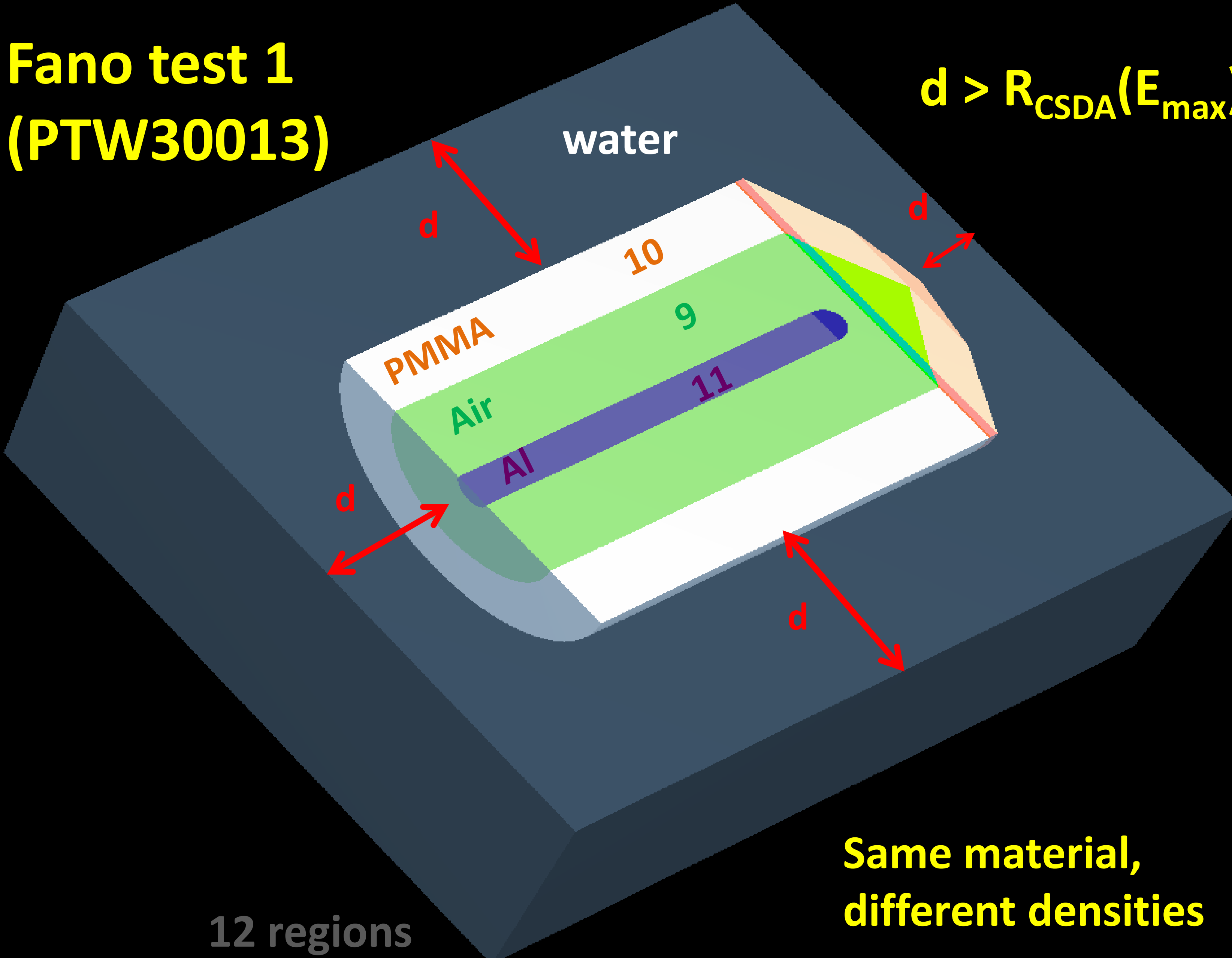


$\rho =$
0.001
0.01
0.1
1
uniform source of electrons,
per unit mass
100 keV
u_0
electron
tracks
in water
phantom


1.01
1.00
0.99
Monte Carlo / Theory
0.001
0.01
0.1
1
0 T
this is what we mean
when we say that
EGSnrc is accurate
at the 0.1% level
in water
phantom

Fano test 1 (PTW30013)

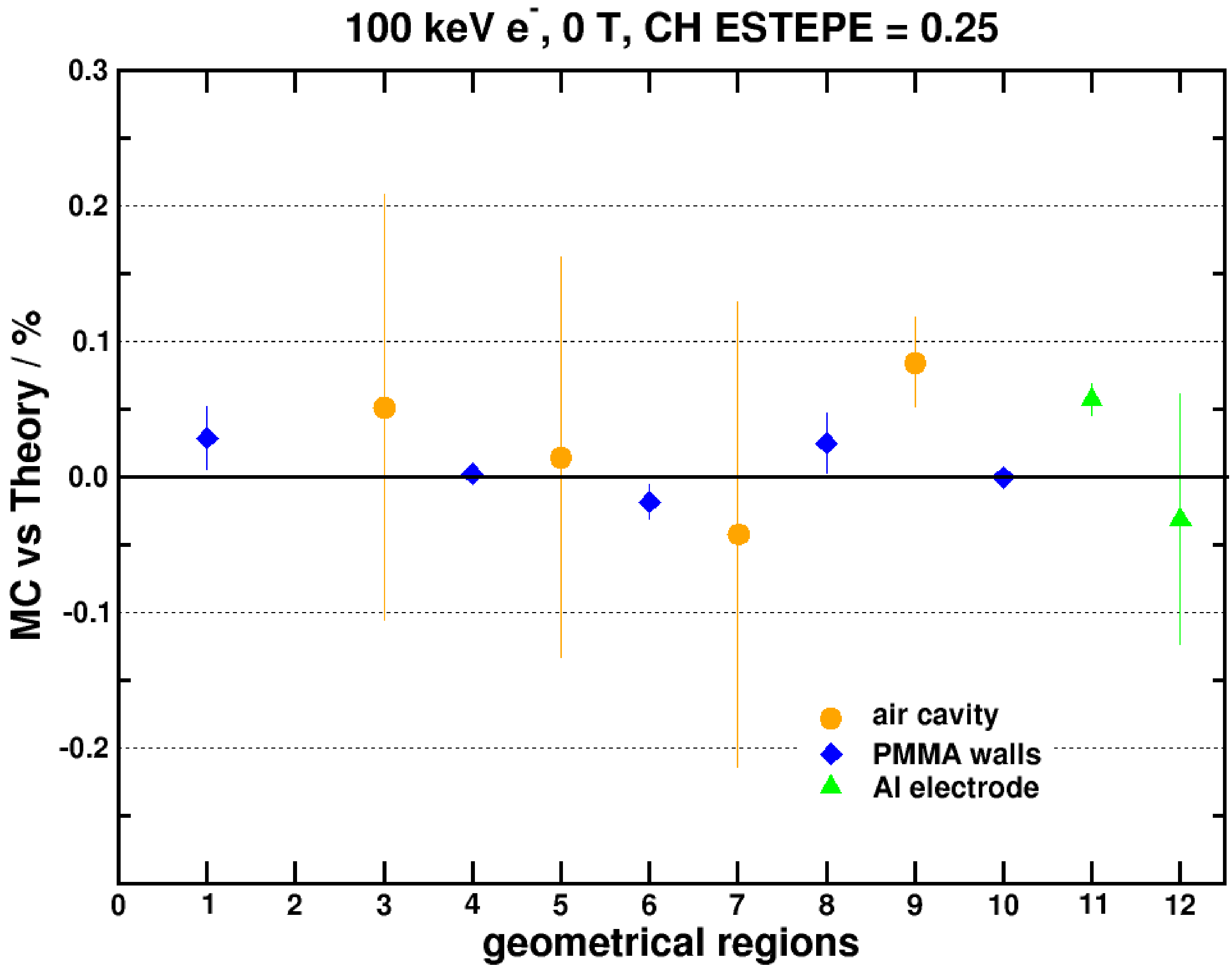
$d > R_{CSDA}(E_{max})$

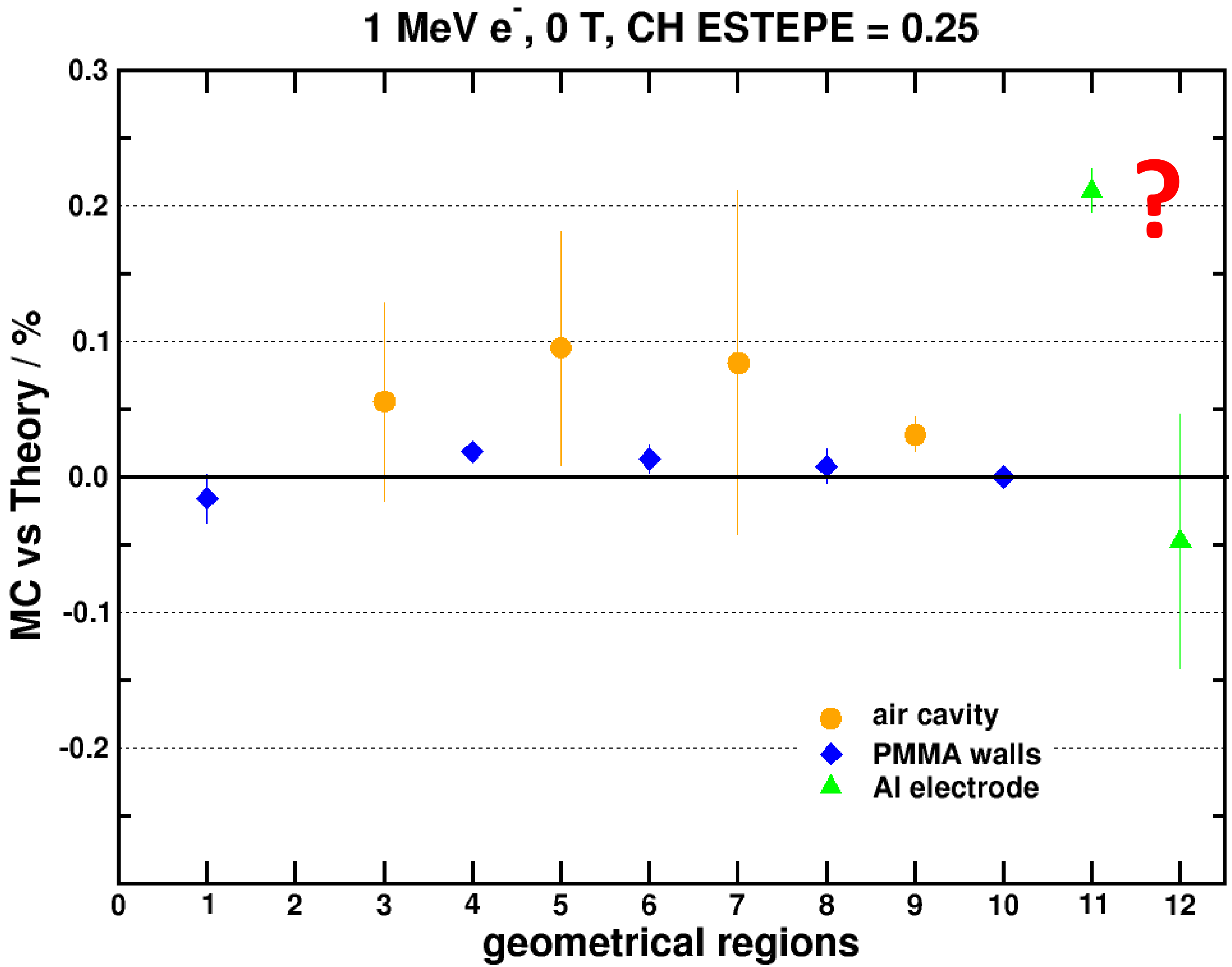


Same material, different densities

12 regions

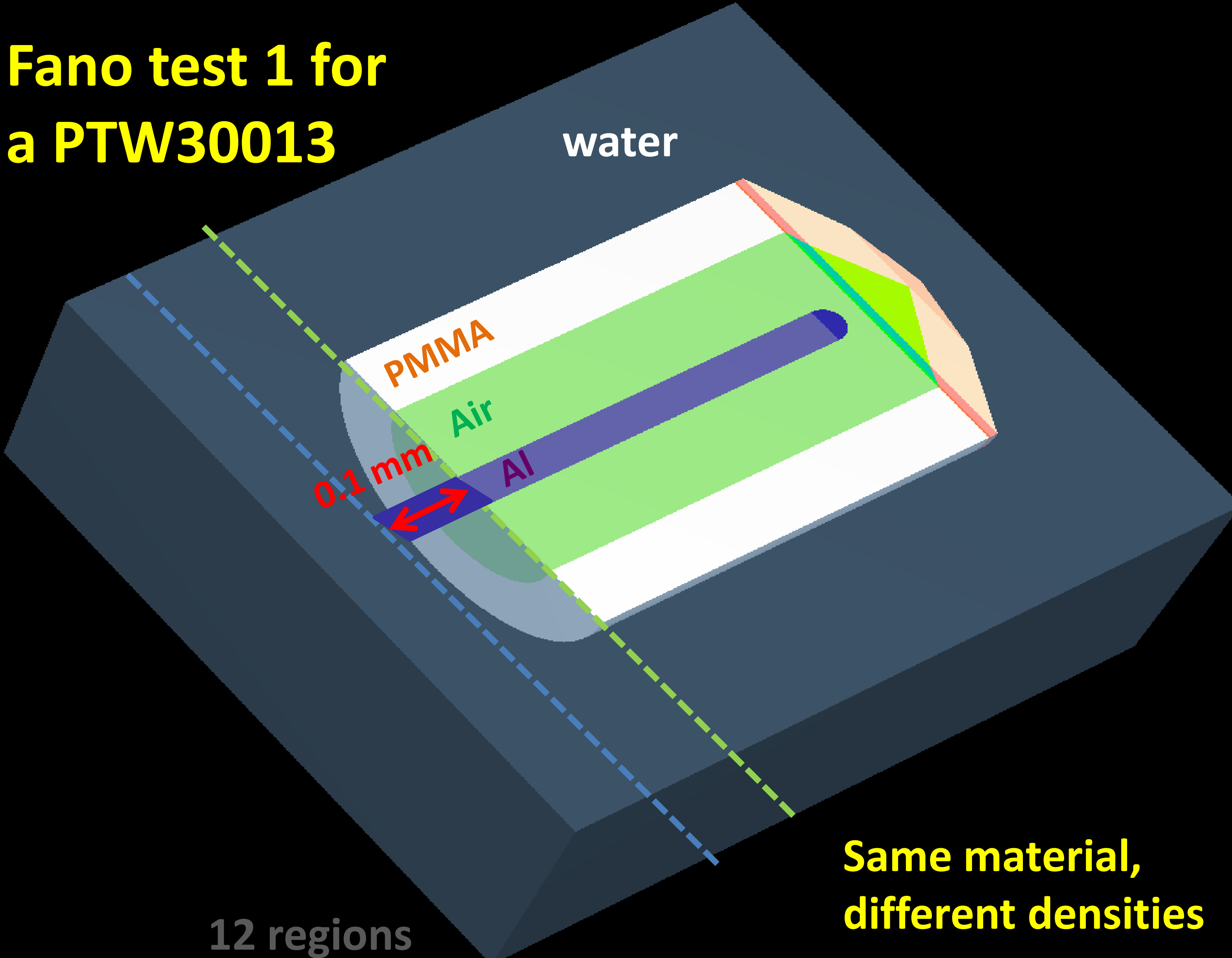
OT


100 keV e⁻, 0 T, CH ESTEPE = 0.25
MC vs Theory / %
0.3
0.2
0.1
0.0
-0.1
-0.2
0
1
2
3
4
5
6
7
8
9
10
11
12
geometrical regions
air cavity
PMMA walls
Al electrode

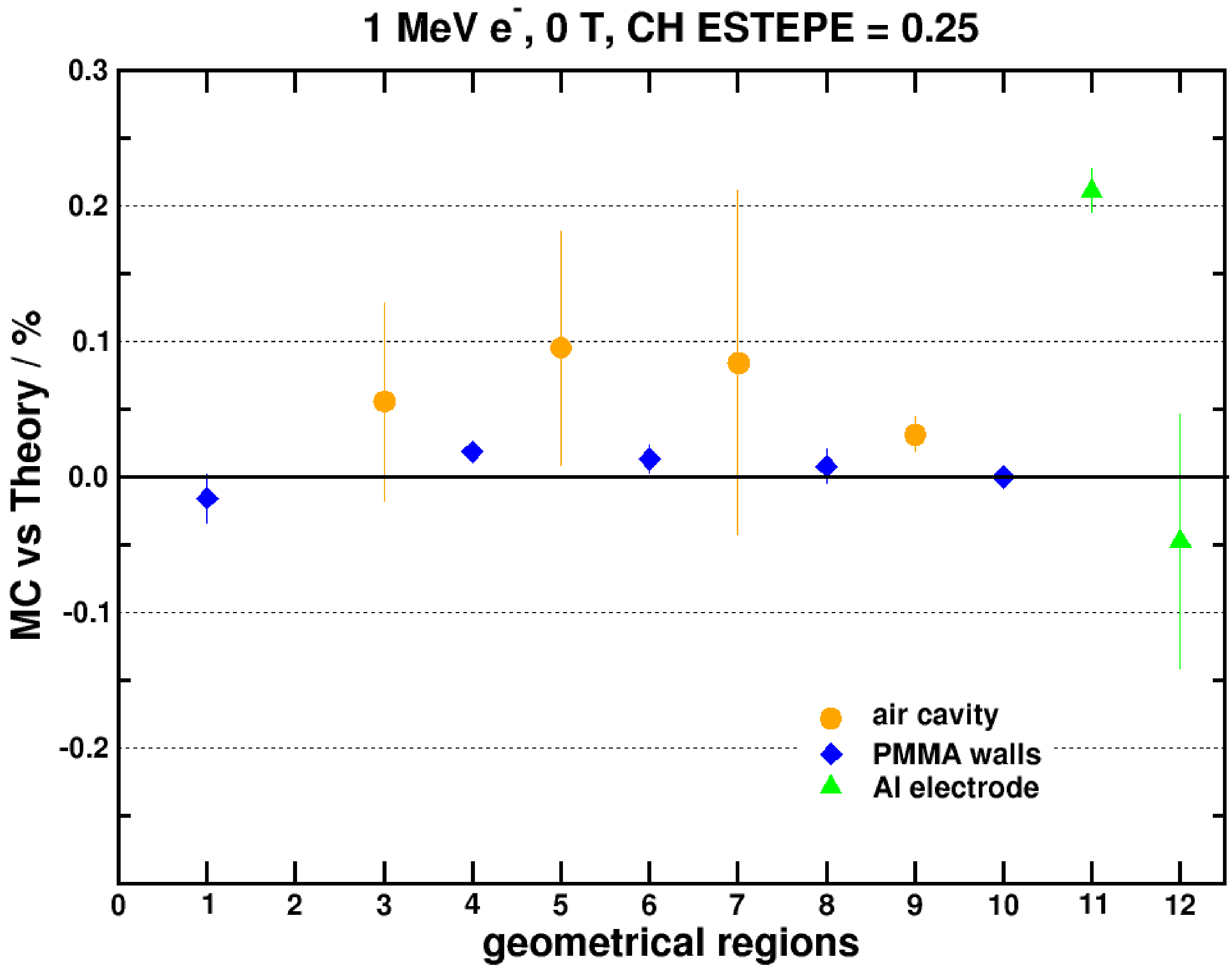


1 MeV e-, 0 T, CH ESTEPE = 0.25
MC vs Theory / %
0.3
0.2
0.1
0.0
-0.1
-0.2
?
0
1
2
3
4
5
6
7
8
9
10
11
12
geometrical regions
air cavity
PMMA walls
Al electrode

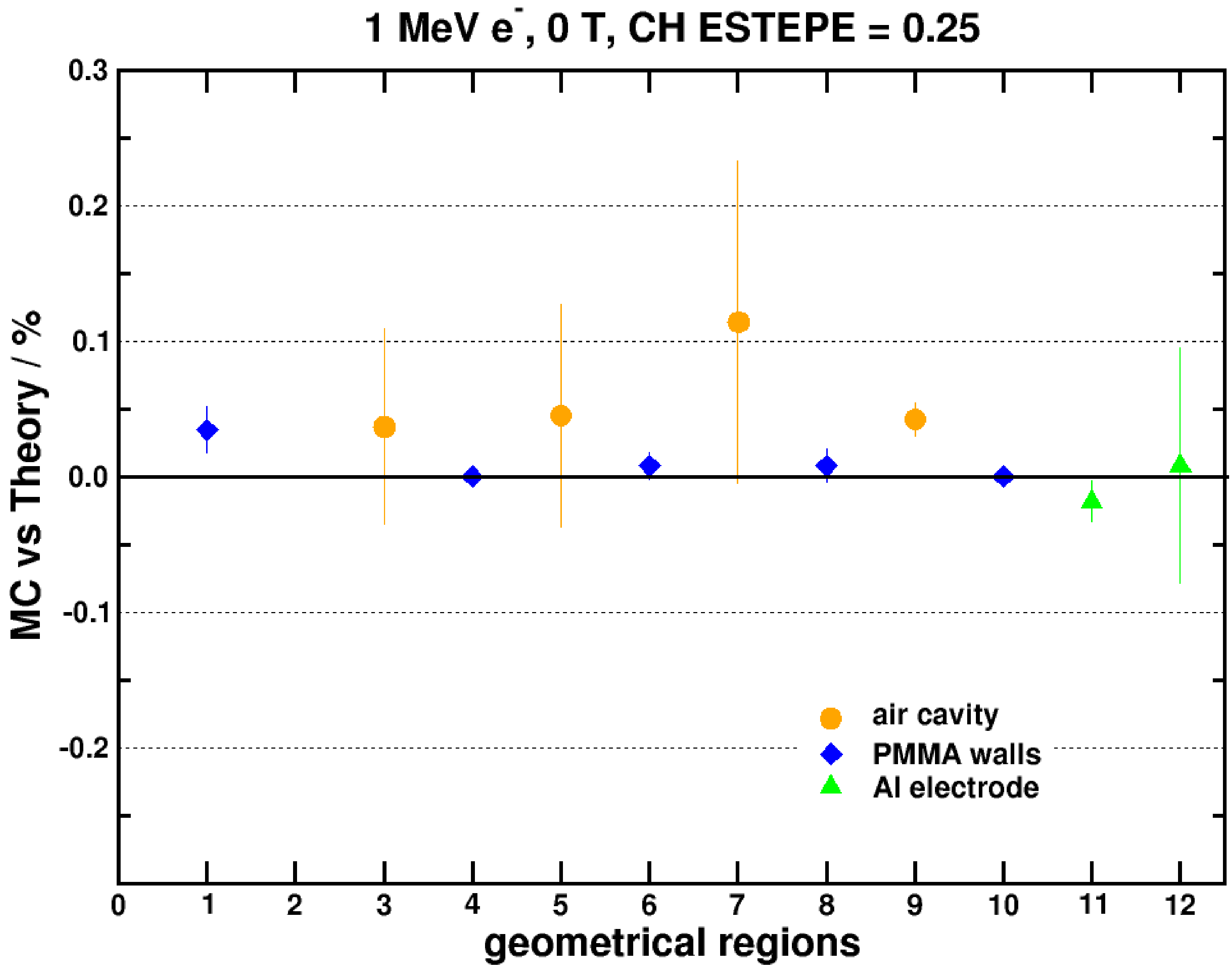
Fano test 1 for a PTW30013



Same material, different densities



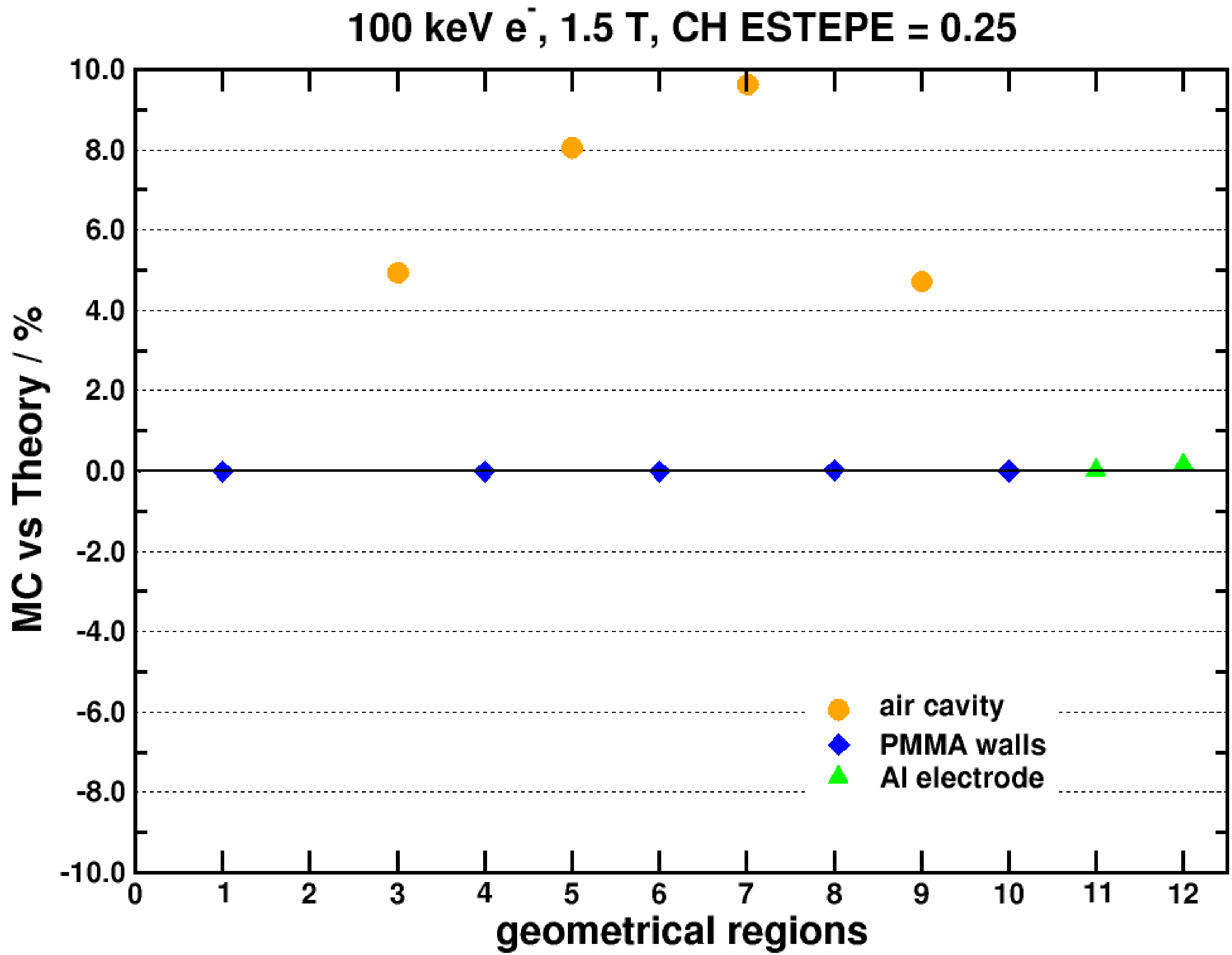
1 MeV e-, 0 T, CH ESTEPE = 0.25
MC vs Theory / %
0.3
0.2
0.1
0.0
-0.1
-0.2
0
1
2
3
4
5
6
7
8
9
10
11
12
geometrical regions
air cavity
PMMA walls
Al electrode



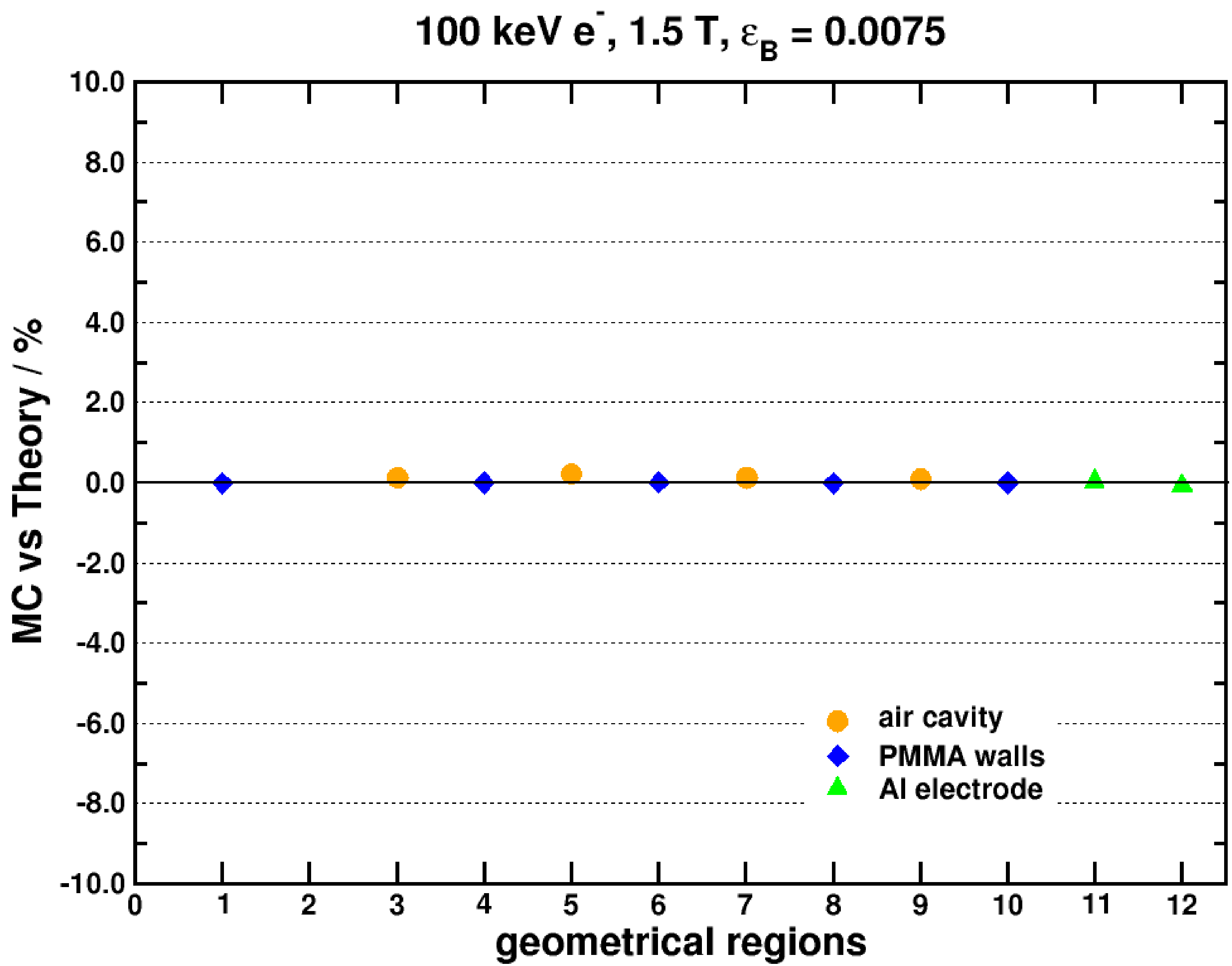
1 MeV e-, 0 T, CH ESTEPE = 0.25
MC vs Theory / %
0.3
0.2
0.1
0.0
-0.1
-0.2
0
1
2
3
4
5
6
7
8
9
10
11
12
geometrical regions
air cavity
PMMA walls
Al electrode

Powerful diagnostic tool !!!

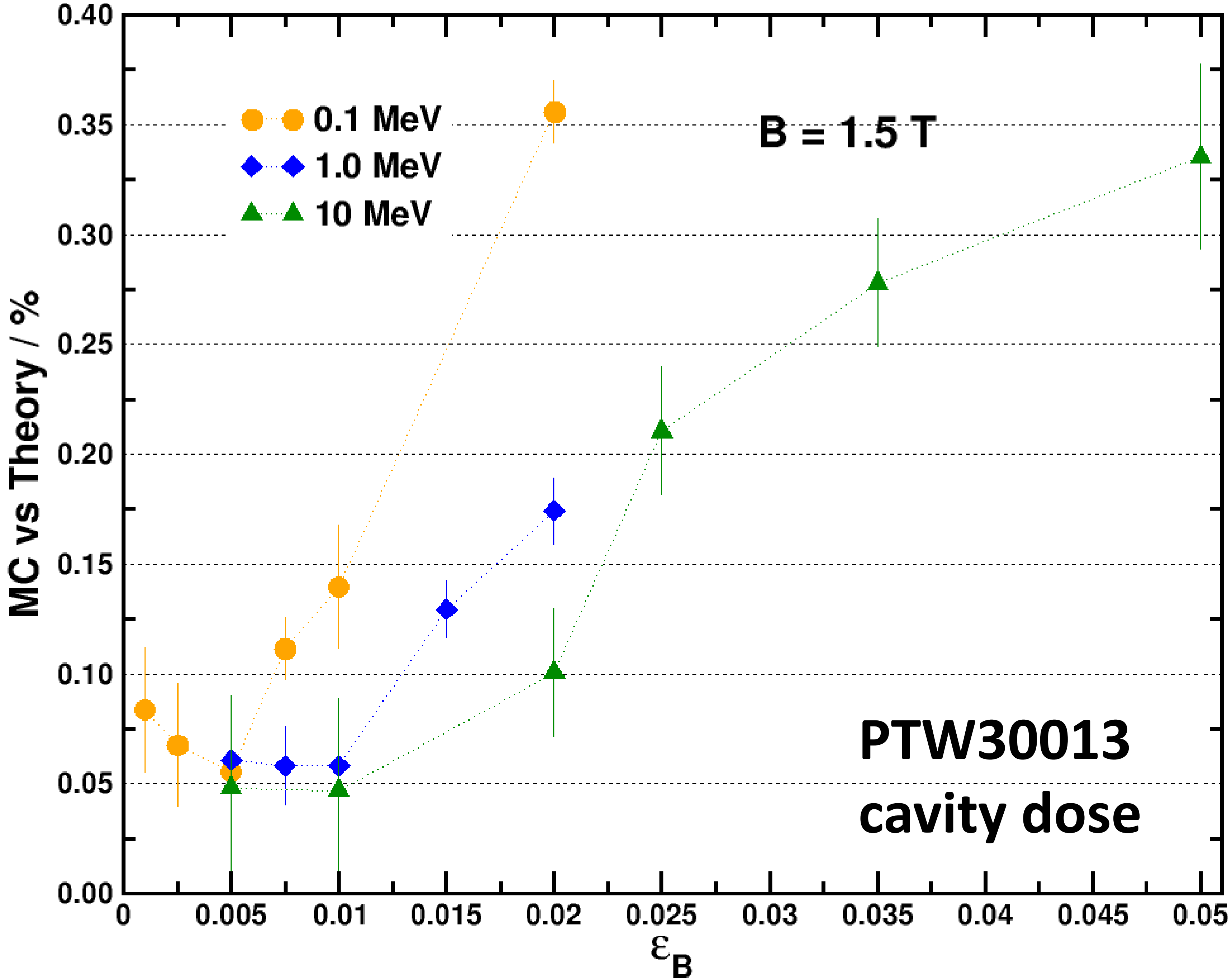
1.5 T



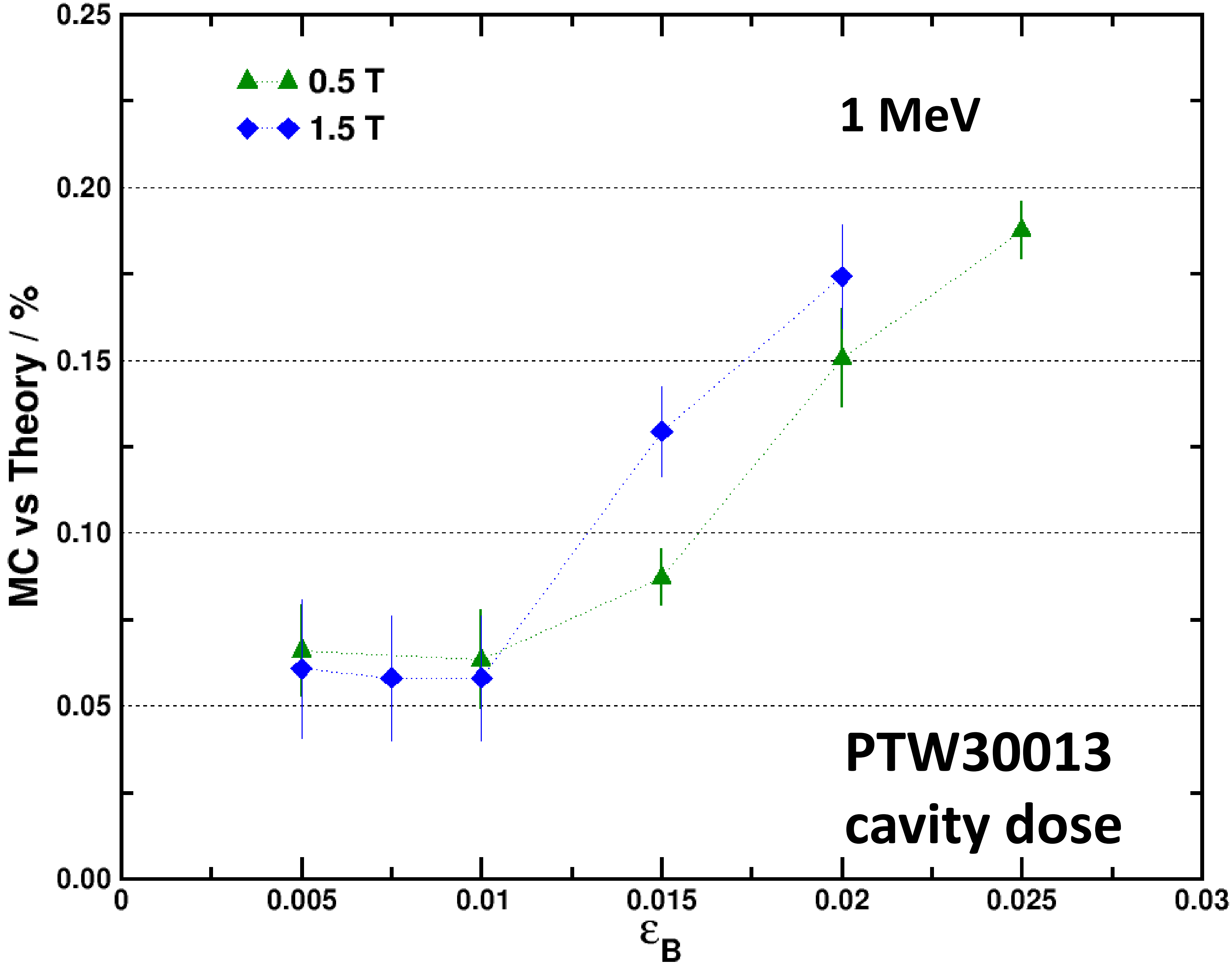
100 keV e⁻, 1.5 T, CH ESTEPE = 0.25
MC vs Theory / %
10.0
8.0
6.0
4.0
2.0
0.0
-2.0
-4.0
-6.0
-8.0
-10.0
0
1
2
3
4
5
6
7
8
9
10
11
12
geometrical regions
air cavity
PMMA walls
Al electrode


100 keV e-, 1.5 T, ε_B = 0.0075
MC vs Theory / %
geometrical regions
air cavity
PMMA walls
Al electrode
10.0
8.0
6.0
4.0
2.0
0.0
-2.0
-4.0
-6.0
-8.0
-10.0
0
1
2
3
4
5
6
7
8
9
10
11
12

Finding ε_B



0.1 MeV
1.0 MeV
10 MeV
B = 1.5 T
PTW30013
cavity dose
MC vs Theory / %
ε_B
0.00
0.05
0.10
0.15
0.20
0.25
0.30
0.35
0.40
0
0.005
0.01
0.015
0.02
0.025
0.03
0.035
0.04
0.045
0.05

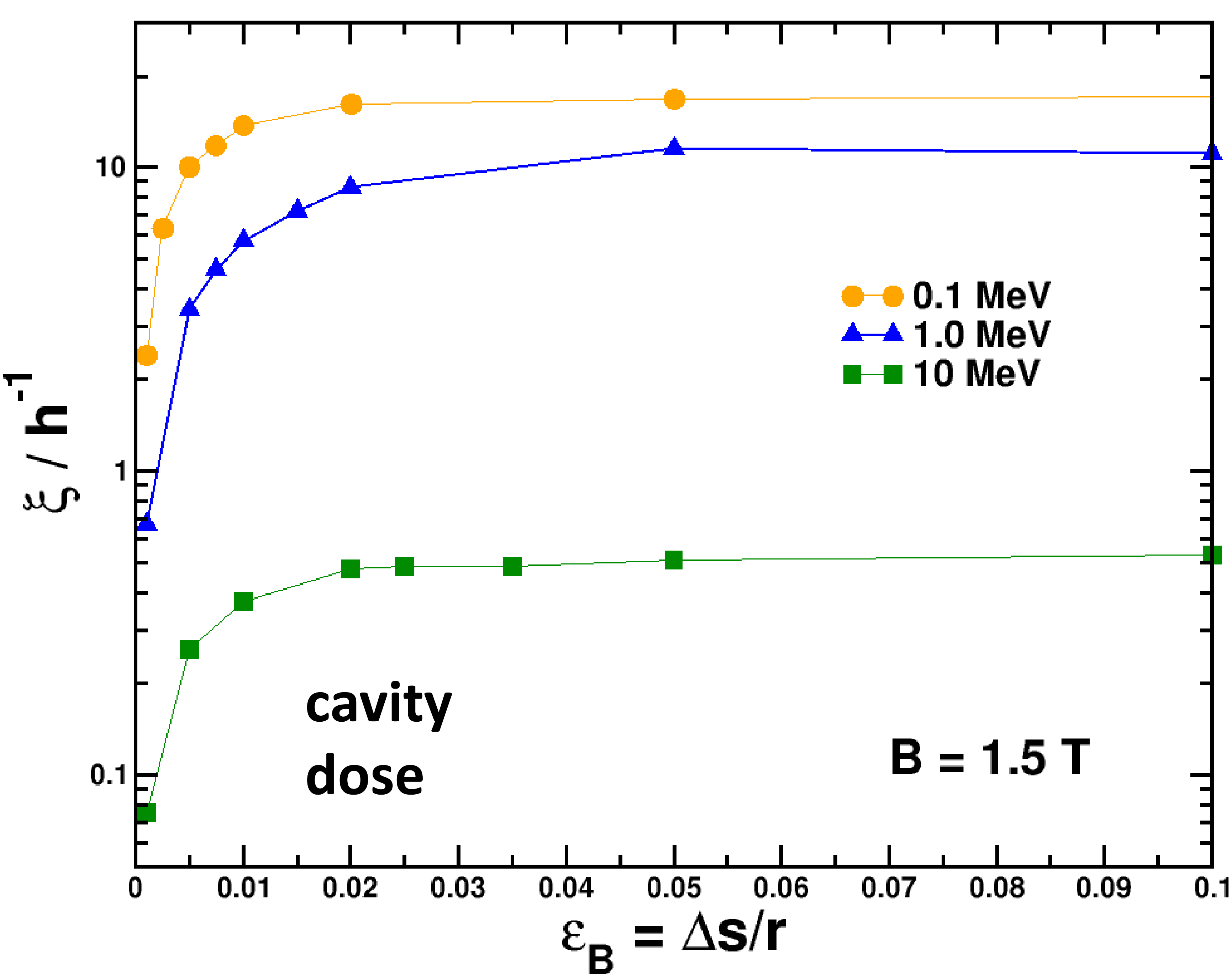

0.5 T
1.5 T
1 MeV
PTW30013
cavity dose
MC vs Theory / %
0.25
0.20
0.15
0.10
0.05
0.00
0
0.005
0.01
0.015
0.02
0.025
0.03
ε_B

Efficiency

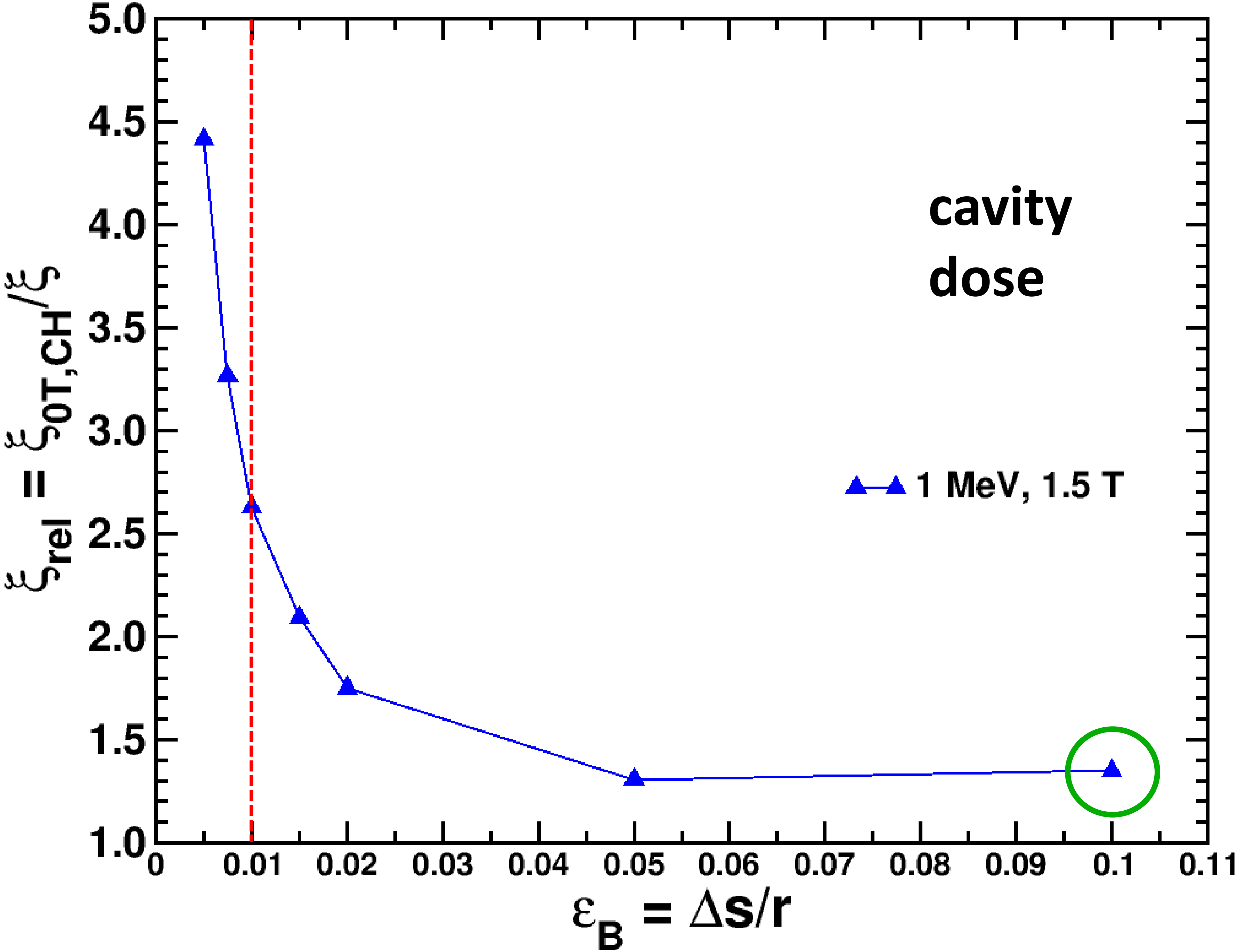
Measuring efficiency

$$\xi = \frac{1}{s^2 \times T_{CPU}}$$

How long needed to achieve desired uncertainty?



0.1 MeV
1.0 MeV
10 MeV
cavity
dose
B = 1.5 T
ξ / h^-1
ε_B = Δs/r
10
1
0.1
0
0.01
0.02
0.03
0.04
0.05
0.06
0.07
0.08
0.09
0.1



cavity
dose
1 MeV, 1.5 T
$\xi_{rel} = \xi_{0T,CH}/\xi$
$\varepsilon_B = \Delta s/r$
5.0
4.5
4.0
3.5
3.0
2.5
2.0
1.5
1.0
0
0.01
0.02
0.03
0.04
0.05
0.06
0.07
0.08
0.09
0.1
0.11

Conclusionss

Transport in electromagnetic field is available in EGSnrc as a first-order correction on the velocity.

Ionization chamber dose response calculations pass Fano test in a magnetic field only with significant step size restrictions.

Larger step sizes are possible as energy increases or field strength decreases (curvature radius increases)

Considering the penalty in efficiency, a more accurate algorithm allowing larger step sizes is desirable.

Fano test: powerful tool for benchmarking radiation transport algorithms and testing the correctness of MC simulation parameters.

NRC·CNRC

Transport under magnetic fields with the EGSnrc simulation toolkit

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Measurement Science and Standards,
National Research Council Canada

Hugo Bouchard

Université de Montréal and
National Physics Laboratory (UK)