

APEX BLIND DECONVOLUTION  
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Good discussion of APEX method in  
**Optical Engineering**, October 2006 issue.

Fourier domain method. Assumes restricted class of blurs. Works on many, but not all, images.

## BLIND DECONVOLUTION.

$$\int_{R^2} h(x-u, y-v) f(u, v) du dv = g(x, y). \quad (1)$$

Find **both** psf  $h(x, y)$  and sharp image  $f(x, y)$ , given blurred image  $g(x, y)$ . **Very difficult.**

APEX Method assumes restricted class of blurs  $h(x, y)$  with Fourier transform given by

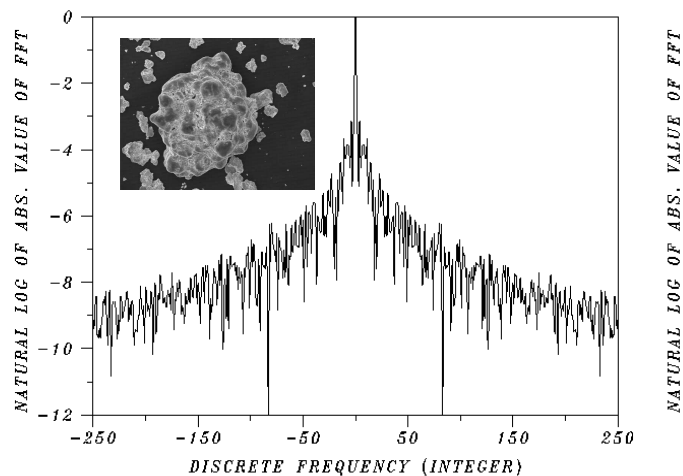
$$\hat{h}(\xi, \eta) = \exp\{-\alpha(\xi^2 + \eta^2)^\beta\}, \quad \alpha > 0, \quad 0 < \beta \leq 1. \quad (2)$$

**Levy stable probability density function.**

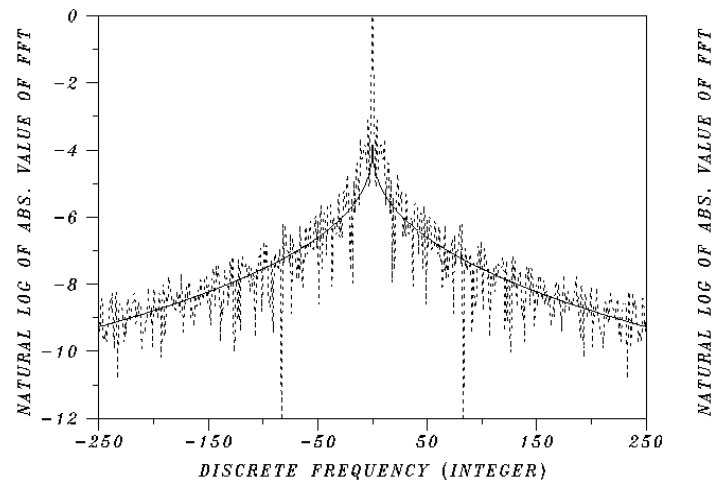
Detects  $(\alpha, \beta)$  by examining asymptotic behavior in Fourier transform of blurred image.

Least squares fit logarithm of blurred image  
 power spectrum with function  $z = -\alpha|\xi|^{2\beta} - A$ .  
 Here, with  $A = 4$ ,  $\Rightarrow \alpha = 0.292$ ,  $\beta = 0.254$ .

Log of power spectrum.



Least squares fit (solid).



Deconvolution by solving fractional diffusion equation **backwards in time**.

$$\frac{\partial u}{\partial t} = -\alpha(-\Delta)^\beta u, \quad 0 < t \leq 1. \quad (3)$$

Very ill-posed. "**Slow Evolution**" (**SECB**) **Constraint**. Solution given in Fourier space.

$$\hat{u}(\xi, \eta, t) = \frac{\hat{h}^t(\xi, \eta) \overline{\hat{h}}(\xi, \eta) \hat{g}(\xi, \eta)}{|\hat{h}(\xi, \eta)|^2 + K^{-2} |1 - \hat{h}^s(\xi, \eta)|^2}, \quad (4)$$

**Regularization parameters**  $s \ll 1, K$ .

Predicted Levy stable  $\hat{h}(\xi, \eta)$



$t = 1$	blurred noisy image $g(x, y)$
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$t = t_\sigma$	SECB regularized solution $u(x, y, t)$ (Partially deblurred image)
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$t = 0$	Fully deblurred image $u(x, y, 0) = f(x, y)$
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**Monitor deconvolution as  $t \downarrow 0$ .**

Calculate diagnostic information

$\| u(t) \|_{L^1}, \quad \| \nabla u(t) \|_{L^1}, \quad \text{as } t \downarrow 0.$

**INITIAL TRIAL PSF TOO WIDE ???**

Good deconvolution requires

$\| u(t) \|_{L^1} = \text{Constant}, \quad \text{as } t \downarrow 0.$

$\| \nabla u(t) \|_{L^1}$  increase monotonically as  $t \downarrow 0$ .

**Enforce  $L^1$  norm conservation.**

Terminate deconvolution at  $t = t_\sigma > 0$ .

And/or readjust initial parameter choices.

Equivalent to readjustment of initial psf  $h(x, y)$ .

## Deconvolution in slow motion.

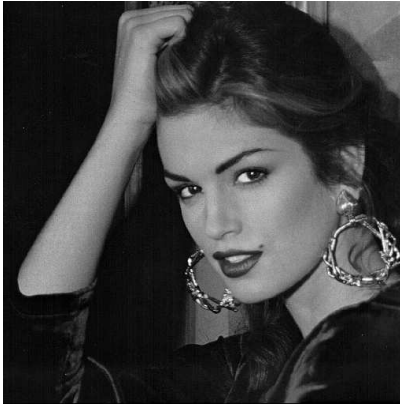
Allows termination **before** reaching  $t = 0$ .

Cindy Crawford sequence. Best at  $t \approx 0.6$ .

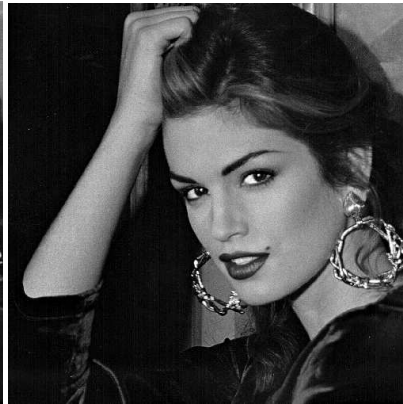
$t=1.0$



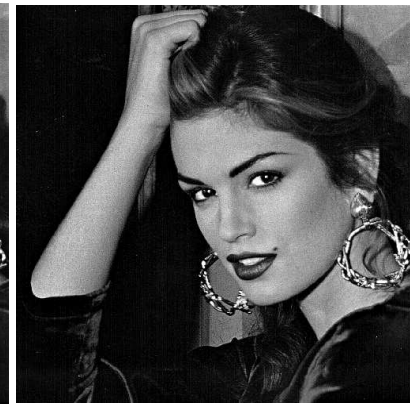
$t=0.8$



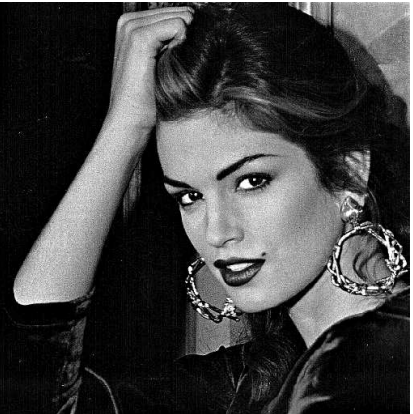
$t=0.7$



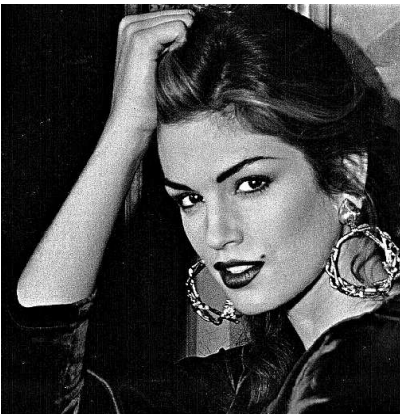
$t=0.6$



$t=0.5$



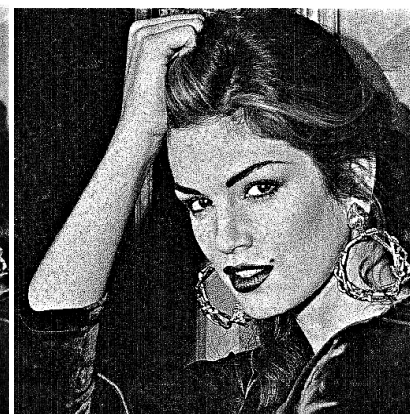
$t=0.4$



$t=0.2$



$t=0.0$

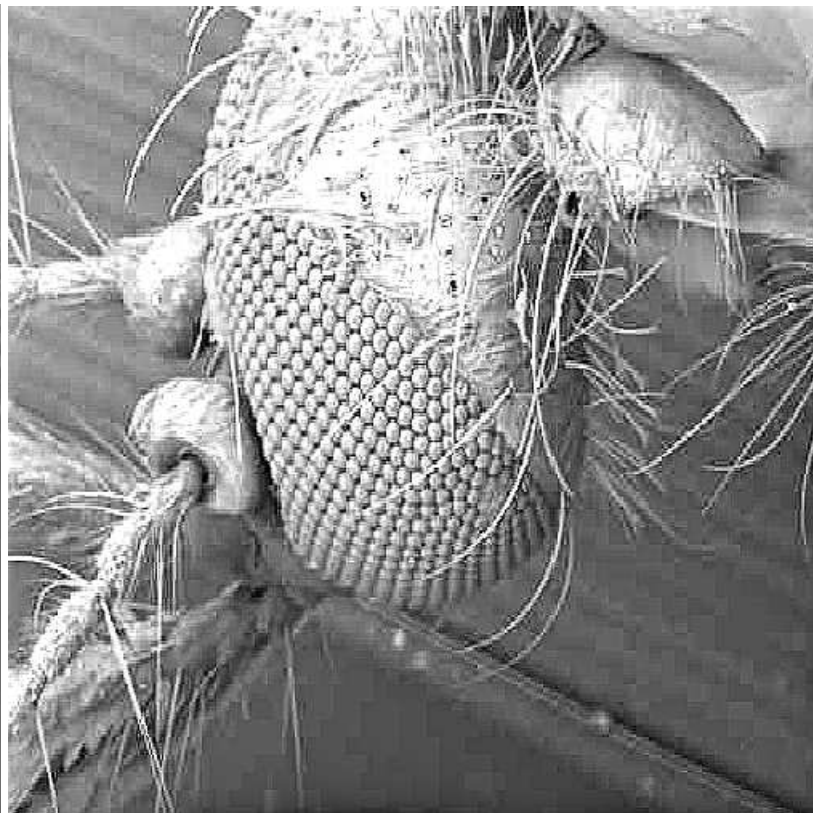


Mosquito's head showing compound eye.

**Original image.**



**After APEX processing.**





## ENGLISH VILLAGE IMAGE.

Original image.



After APEX processing.



Zooming now reveals Holstein cows in meadow.

**Zoom in Original.**



**Zoom in APEX.**



## SHARPENING MEDICAL IMAGES.

APEX BLIND DECONVOLUTION OF REAL SAGITTAL BRAIN MRI.

Original Brain MRI.



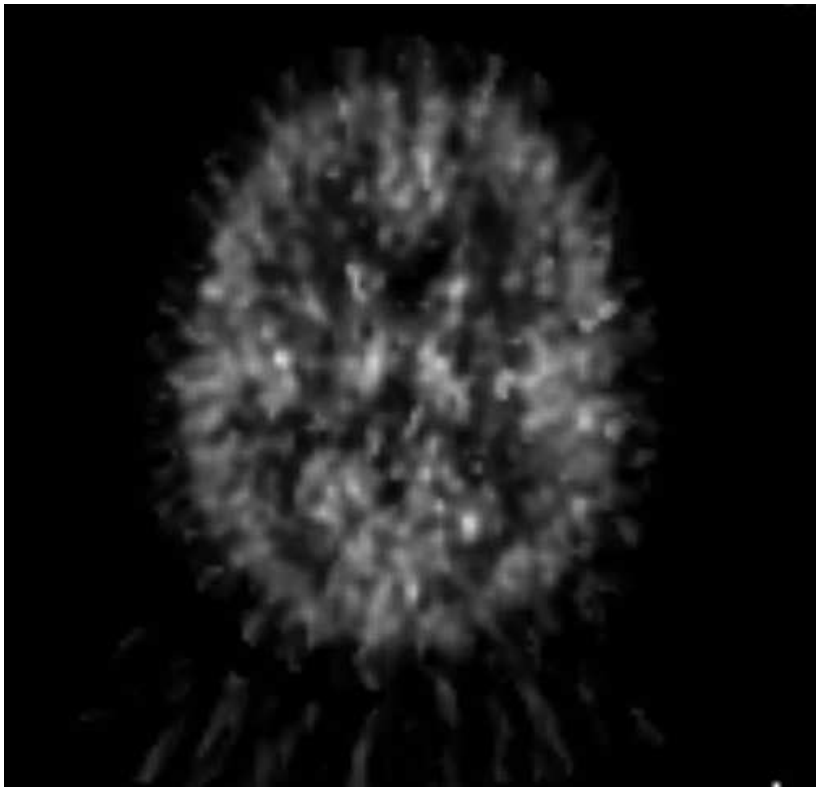
After APEX processing.



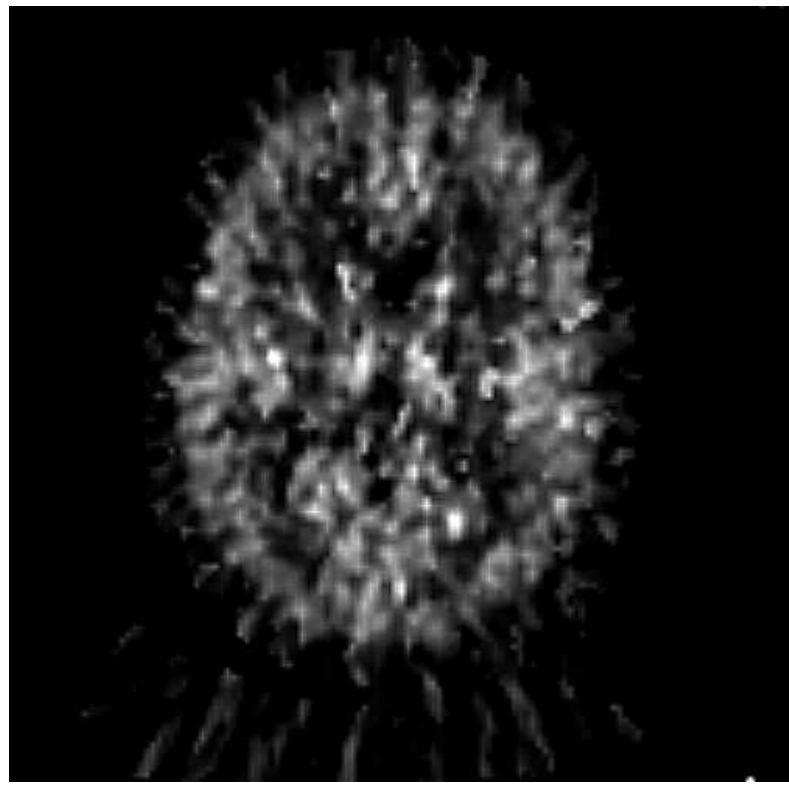
Functional PET imaging.

Transverse 128 × 128 brain slices.

Original PET slice.

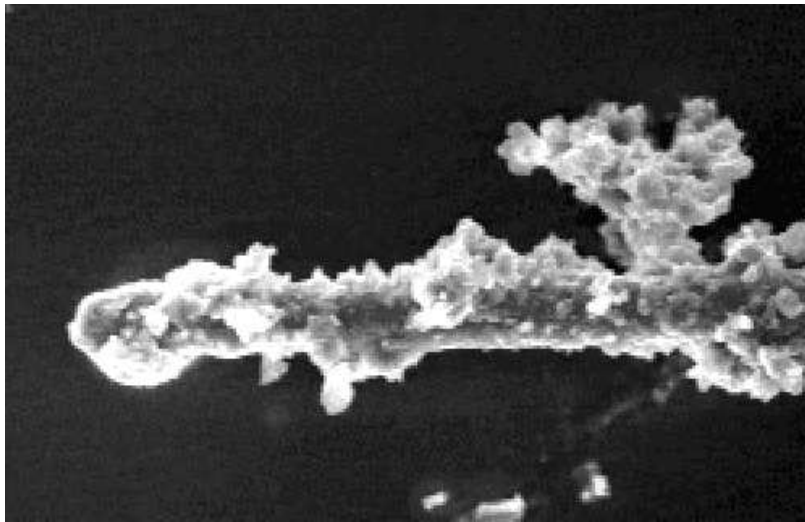


After APEX processing.

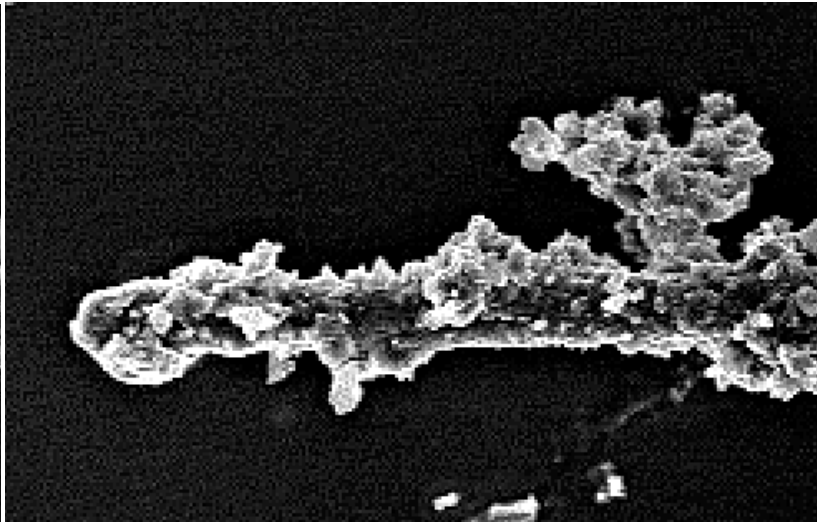


NIST Nanoscale Scanning Electron Microscopy.  
(with David Bright and Andras Vladar)

**Original image.**

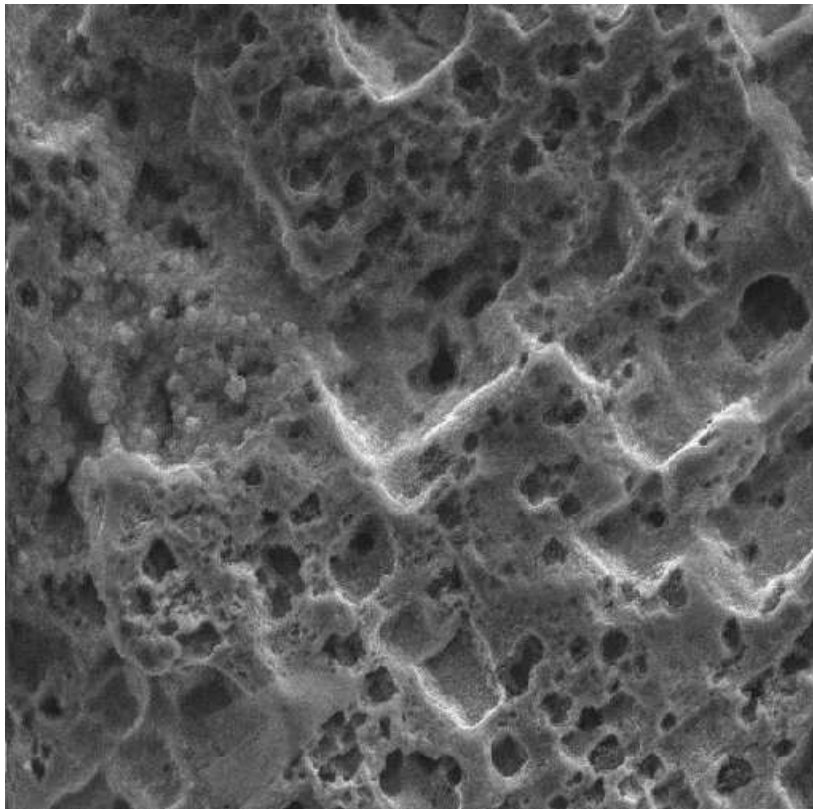


**After APEX processing.**

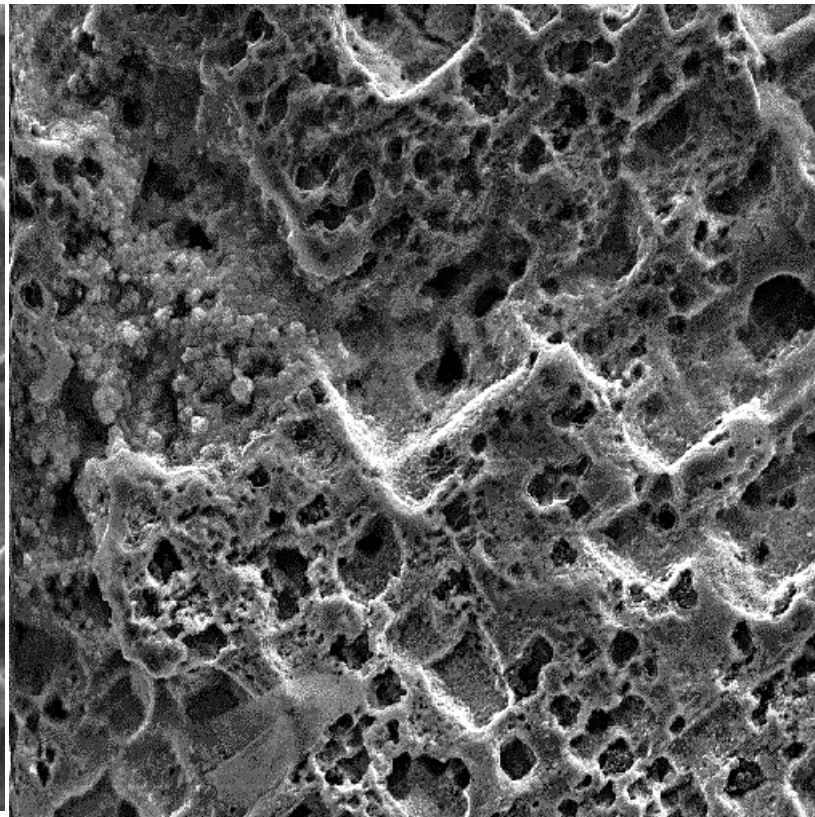


## NIST Nanoscale Scanning Electron Microscopy.

**Original image.**



**After APEX processing.**





HUBBLE TELESCOPE TADPOLE GALAXY  
Background of considerable interest. "Whitman's Sampler of galaxies stretching back to beginning of time....."

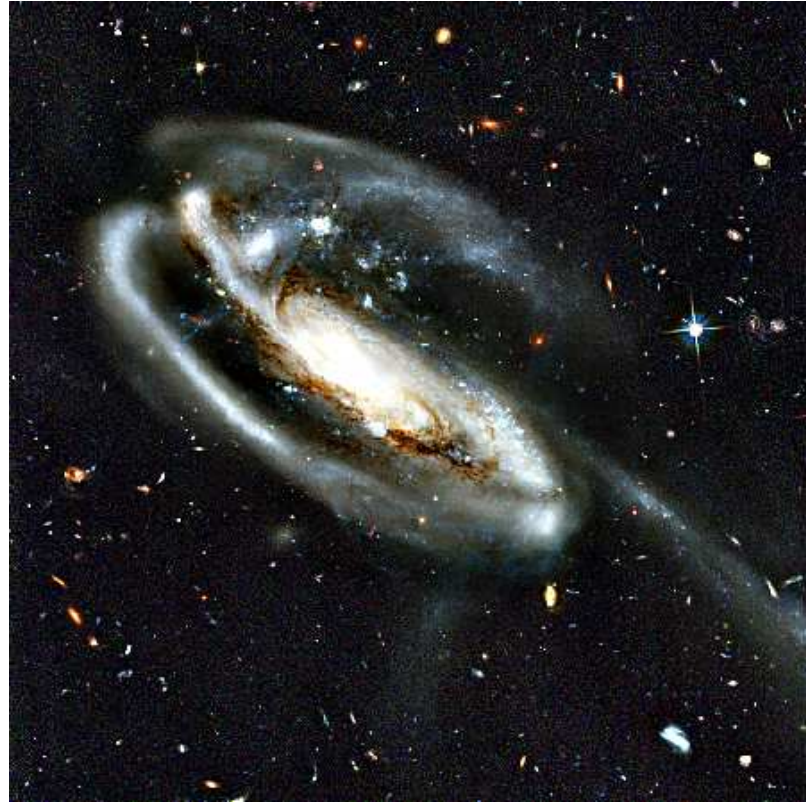


Zooming on Tadpole head reveals fine scale structure, and more background galaxies....  
Original taken with NASA's sophisticated  
'Advanced Camera for Surveys' (ACS).

**Original Hubble image.**



**After APEX processing.**





APEX-detected otfs very far from Gaussian.  
Not previously known in imaging science.  
Mysterious origin. Significant discovery ??

